



The cosmological constant derived via galaxy groups and clusters

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Received: 5 February 2019 / Accepted: 14 February 2019 / Published online: 25 February 2019
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Abstract The common nature of dark matter and dark energy is argued in Gurzadyan (Eur Phys J Plus 134:14, 2019) based on the approach that the cosmological constant Λ enters the weak-field General Relativity following from Newton theorem on the “sphere-point mass” equivalency (Gurzadyan and Stepanian in Eur Phys J C 78:632, 2018). Here we probe the Λ -gravity description of dark matter in galaxy systems, from pairs up to galaxy clusters using the data of various sources, i.e. of Local Supercluster galaxy surveys, gravity lensing and Planck satellite. The prediction that the cosmological constant has to be the lower limit for the weak-field Λ obtained from galaxy systems of various degree of virialization is shown to be supported by those observations. The results therefore support the Λ -gravity nature of dark matter in the studied systems, implying that the positivity of the cosmological constant might be deduced decades ago from the dynamics of galaxies and galaxy clusters far before the cosmological SN surveys.

1 Introduction

A number of approaches are considered to reveal the nature of dark matter, including prediction of exotic particles and modified gravity models; for review see [3]. One of the recent approaches [1,2] is based on the General Relativity (GR) with a modified weak field limit following from the Newton theorem on equivalency of gravity of sphere and of point mass. That approach enables the common description of dark matter and dark energy, where Λ acts as a universal constant defining the GR and its weak field limit, along with the gravitational constant G [2]. An observational test for that approach is suggested involving the effect of gravity lensing [4].

The notable point following from the Newton theorem is that, on the one hand, Λ acts as a cosmological constant

describing the expansion of the Universe, on the other hand, the same it defines the weak field gravity proper for the dynamics of galaxies, galaxy groups and clusters, i.e. on the distance scales where the Hubble flow cannot be valid. As shown in [1] the value of the cosmological constant indeed is supported by the parameters of galactic halos and of a sample of galaxy groups.

Here we continue to address the principal issue on the Λ -nature of dark matter considering the data on galaxy systems, from galaxy pairs to galaxy clusters. Among the used samples are data on galaxy groups of Local Supercluster, of galaxy clusters obtained both with gravity lensing and by Planck satellite. Assuming that the dynamical structure of the considered galaxy systems is governed by the modified Newton law [1], we obtain Λ which appear limited from below by the value of cosmological constant. In other words, the value of the cosmological constant is extracted from galaxy systems and not from cosmology.

2 The Λ constant and galaxy systems

The Newton theorem on “sphere-point” equivalency enables one to arrive at the GR metric [1,2]

$$g_{00} = 1 - \frac{2Gm}{rc^2} - \frac{\Lambda r^2}{3}; \quad g_{rr} = 1 + \frac{2Gm}{rc^2} + \frac{\Lambda r^2}{3}. \quad (1)$$

This implies that the weak field limit for GR as modified Newton law involves two constants (for the potential)

$$\phi(r) = C_1 r^{-1} + C_2 r^2, \quad (2)$$

where C_1 is assigned to G and C_2 to Λ (within numerical factors).

Equation (1) was known before as Schwarzschild – de Sitter metric [5], where the constant Λ was introduced by Einstein to describe the static cosmological model.

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Table 1 Background geometries for vacuum solutions

	Spacetime	Isometry group
$\Lambda > 0$	de Sitter	O(1,4)
$\Lambda = 0$	Minkowski	IO(1,3)
$\Lambda < 0$	Anti de Sitter	O(2,3)

Within our approach Λ is emerging from the general function satisfying the Newton’s theorem and hence naturally emerges in weak field GR, and that correspondence can be represented via isometry groups. Namely, the Lorentz group O(1,3) acts as stabilizer subgroup of isometry group of 4D maximally symmetric Lorentzian geometries [2]. The results are shown in Table 1 and for all these geometries the Lorentzian spheres can be considered as points. Consequently, depending on the sign of Λ , we will have the following non-relativistic limits

$$\begin{aligned}
 O(1, 4) &\rightarrow (O(3) \times O(1, 1)) \ltimes R^6, \\
 O(2, 3) &\rightarrow (O(3) \times O(2)) \ltimes R^6, \\
 IO(1, 3) &\rightarrow (O(3) \times R) \ltimes R^6,
 \end{aligned}
 \tag{3}$$

for all three cases, since the spatial algebra is Euclidean i.e.

$$E(3) = R^3 \ltimes O(3). \tag{4}$$

Hence, for all three cases introduced above the O(3) is the stabilizer group for spatial geometry. This conclusion, in its turn, can be considered as the Newton’s theorem in the language of group theory.

Then, as shown in [1], the weak field Λ extracted from galactic halo and galaxy group data quantitatively coincides with the Λ obtained from cosmological data.

The value of Λ as of the cosmological constant is currently obtained by several observational methods, the one obtained by Planck data yields [6]

$$\Lambda_{PL} = 1.11 \cdot 10^{-52} \text{ m}^{-2}. \tag{5}$$

Table 2 SIS profile

Subsamples	σ (Km/s)	$M_{200} (M_{\odot})$	$\Lambda (m^{-2})$
Total Sample	223 ± 24	(6.9 ± 2.2)E12	5.64E−51
Non Interacting	200 ± 38	(5.0 ± 3.0)E12	5.62E−51
Interacting Pairs	237 ± 29	(8.3 ± 3.0)E12	5.63E−51
Red Pairs	264 ± 28	(11.4 ± 3.7)E12	5.65E−51
Blue Pairs	167 ± 45	(2.9 ± 2.4)E12	5.64E−51
Higher Luminosity Pairs	278 ± 27	(13.2 ± 3.8)E12	5.69E−51
Lower Luminosity Pairs	149 ± 50	(2.0 ± 2.0)E12	5.75E−51
Average			5.66E−51
SD			4.21E−53

For Λ -modified Newtonian gravity Eq. (2) the following relation for virialized systems was derived [1]

$$\Lambda = \frac{3\sigma^2}{2c^2 R^2}, \tag{6}$$

where R is system’s radius and σ is the velocity dispersion.

To test the conclusion on Λ -nature of dark matter in galaxy systems, below we use Eq. (6) for the analysis of data samples on galaxy pairs, galaxy groups and galaxy clusters.

2.1 Galaxy pairs

To probe Eqs. (1) and (2) for galaxy systems we start with galaxy pairs. Here, using the weak lensing data by Gonzalez et al. [7], we estimate the value of Λ from Eq. (6) as shown in Tables 2 and 3 for given classes of subsamples. Estimations are done both for Singular Isothermal sphere (SIS) and Navarro–Frenk–White (NFW) models as described in details in [7], M_{200} corresponds the mass (for the given model) within radius R_{200} , when the mean density equals 200 critical density values of the universe.

2.2 Galaxy groups

Data on galaxy groups as of systems containing 3 and more galaxies situated within the Local Supercluster, namely, of the Leo/Cancer region and Bootes strip are obtained by Karachentsev et al. [8,9]; the analysis of their data on groups of the Hercules-Bootes region [10] is given in [1]. Tables 4 and 5 contain the observational data i.e. the velocity dispersion of galaxies σ and the harmonic average radius R_h of the groups denoted by their brightest galaxy listed in the first column. The last column contains Λ estimated by Eq. (6).

2.3 Galaxy clusters

We now turn to galaxy clusters, the higher scale structure in the hierarchy of galaxy systems. We use the data both on cluster gravity lensing and supernova survey with HST

Table 3 NFW profile

Subsamples	R ₂₀₀ (Mpc)	M ₂₀₀ (M _⊙)	Λ (m ⁻²)
Total sample	0.30 ± 0.03	(7.1 ± 2.1)E12	2.00E-50
Non interacting	0.27 ± 0.05	(5.2 ± 2.8)E12	2.01E-50
Interacting pairs	0.32 ± 0.03	(8.0 ± 2.9)E12	1.86E-50
Red pairs	0.36 ± 0.03	(12.1 ± 3.6)E12	1.97E-50
Blue pairs	0.22 ± 0.06	(2.7 ± 2.1)E12	1.93E-50
Higher luminosity pairs	0.36 ± 0.03	(12.7 ± 3.8)E12	2.07E-50
Lower luminosity pairs	0.19 ± 0.07	(1.7 ± 1.8)E12	1.89E-50
Average			1.96E-50
SD			6.77E-52

(CLASH) [11]. In the Table 6 we exhibit the data for all 19 clusters of CLASH survey along with the results of estimation of Λ using Eq. (6).

Besides the CLASH data we use also data of 3 Planck clusters from [12] and represent the estimated Λ (Table 7).

Finally, we estimate the Λ for a sample of Local Cluster Substructure Survey (LoCuSS) clusters [13], as represented in Table 8.

The advantages of using CLASH data is that they provide the values for virial masses M_{vir} of clusters. Although the degree of virialization of a given cluster varies from one cluster to another and hence is a separate issue for analysis, those masses provide the upper limit of error for the value of Λ.

Namely, by considering the Λ-modified potential from one side, and the error limit for M_{vir}, on the other side, we have an upper limit for the value of Λ

$$\frac{\mathbb{E}(\sigma^2)}{\sigma^2} = \frac{\Lambda R_{vir}^3 c^2}{6GM_{vir}}, \tag{7}$$

where $\mathbb{E}(\sigma^2)$ is the error limit for σ^2 . The results for the CLASH clusters are shown in Table 9. Note that, for all these clusters the error limit covers the value of Λ as of cosmological constant.

Table 4 Galaxy groups of Leo/Cancer region

Group	σ _v (km/s ⁻¹)	R _h kpc	Λ(m ⁻²)
NGC2648	55	128	3.24E-51
NGC2775	89	296	1.59E-51
NGC2894	50	458	2.09E-52
NGC2962	53	161	1.90E-51
NGC2967	62	507	2.63E-52
UGC5228	40	188	7.95E-52
NGC3023	21	35	6.32E-51
NGC3020	45	44	1.84E-50
NGC3049	15	144	1.91E-52
UGC5376	66	253	1.20E-51
NGC3166	44	126	2.14E-51
NGC3227	74	128	5.87E-51
NGC3338	50	112	3.50E-51
NGC3379	193	191	1.79E-50
NGC3423	21	570	2.38E-53
NGC3521	37	132	1.38E-51
NGC3596	42	41	1.84E-50
NGC3607	115	471	1.05E-51
NGC3626	86	187	3.72E-51
NGC3627	136	201	8.04E-51
NGC3640	134	252	4.97E-51
NGC3686	91	175	4.75E-51
NGC3810	43	360	2.51E-52
Average			4.62E-51
SD			5.70E-51

Table 5 Galaxy groups of Bootes strip region

Group	σ _v (km/s ⁻¹)	R _h (kpc)	Λ(m ⁻²)
N4900	36	116	1.69E-51
N5248	38	151	1.11E-51
N5363	114	165	8.39E-51
N5506	23	35	7.59E-51
N5566	103	196	4.85E-51
N5638	74	203	2.33E-51
P51971	10	100	1.76E-52
IC1048	83	150	5.38E-51
N5746	107	296	2.30E-51
N5775	87	120	9.23E-51
N5792	48	290	4.81E-52
N5838	53	210	1.12E-51
N5846	228	415	5.30E-51
Average			3.84E-51
SD			3.01E-51

Table 6 CLASH Survey

Cluster	R_{vir} (Mpc)	M_{vir} (M_{\odot})	Λ (m^{-2})
Abell 383	1.86	(1.04 ± 0.07)E15	1.23E−50
Abell 209	1.95	(1.17 ± 0.07)E15	1.20E−50
Abell 2261	2.26	(1.76 ± 0.18)E15	1.16E−50
RXJ2129+0005	1.65	(0.73 ± 0.07)E15	1.24E−50
Abell 611	1.79	(1.03 ± 0.07)E15	1.37E−50
MS2137-2353	1.89	(1.26 ± 0.06)E15	1.42E−50
RXCJ2248-4431	1.92	(1.40 ± 0.12)E15	1.51E−50
MACSJ1115+0129	1.78	(1.13 ± 0.10)E15	1.52E−50
MACSJ1931-26	1.61	(0.83 ± 0.06)E15	1.51E−50
RXJ1532.8+3021	1.47	(0.64 ± 0.09)E15	1.53E−50
MACSJ1720+3536	1.61	(0.88 ± 0.08)E15	1.60E−50
MACSJ0429-02	1.65	(0.96 ± 0.14)E15	1.63E−50
MACSJ1206-08	1.63	(1.00 ± 0.11)E15	1.76E−50
MACSJ0329-02	1.54	(0.86 ± 0.11)E15	1.79E−50
RXJ1347-1145	1.80	(1.35 ± 0.19)E15	1.76E−50
MACSJ1311-03	1.28	(0.53 ± 0.04)E15	1.92E−50
MACSJ1423+24	1.34	(0.65 ± 0.11)E15	2.06E−50
MACSJ0744+39	1.33	(0.79 ± 0.04)E15	2.56E−50
CLJ1226+3332	1.57	(1.72 ± 0.11)E15	3.38E−50
Average			1.69E−50
SD			5.16E−51

Meantime, let us note that the virial mass and radius can be defined as [14]

$$M = \frac{4}{3}\pi R^3 \Delta_c \rho_{crit}, \tag{8}$$

where Δ_c is the overdensity parameter and ρ_{crit} is the critical density of the universe. The Δ_c , in its turn is obtained for each era of the universe as [15]

$$\Delta_c = 18\pi^2 + 82x - 39x^2, \quad x = \Omega(z) - 1, \\ \Omega(z) = \frac{H_0^2}{H^2(z)} \Omega_0 (1+z)^3, \tag{9}$$

where H is the Hubble constant.

Thus it turns out that considering the Newtonian virial theorem $\sigma^2 = \frac{GM_{vir}}{R_{vir}}$ and Eq. (8) for virialized systems one

can calculate the value of Λ based on Eq. (6)

$$\Lambda = \frac{3}{4} \frac{\Delta_c}{c^2} H^2. \tag{10}$$

Eq. (9) enables one to calculate the value of Δ_c at each era of Universe. For pure de Sitter universe $\Delta_c = 18\pi^2 \approx 178$, while within the “spherical collapse” model its value yields $\Delta_c = 200$ [16]. However, within the Λ CDM cosmology $\Delta_c \approx 100$ ([17] and references therein). Considering all above, for $H = 70$ (km/s)/(Mpc) one will obtain $\Lambda \approx 4.30 \cdot 10^{-51} m^{-2}$.

3 Conclusions

We studied the relevance of the weak field GR with modified potential Eq. (2) to the description of the dynamics of

Table 7 Planck clusters

Clusters	σ (Km/s)	M_{200} (M_{\odot})	Λ (m^{-2})
PSZ1 G109.88+27.94	1800 ± 200	(44 ⁺¹⁶ ₋₁₃)E14	4.96E−51
PSZ1 G139.61+24.20	800 ± 100	(6.3 ^{+2.7} _{-2.1})E14	3.58E−51
PSZ1 G186.98+38.66	1100 ± 200	(14.5 ^{+9.4} _{-6.5})E14	3.88E−51
Average			4.14E−51
SD			5.93E−52

Table 8 LoCuSS Clusters

Cluster	$R_{200} (h_{100}^{-1} Mpc)$	$M_{200} (M_{\odot})$	$\Lambda (m^{-2})$
Abell 586	1.2 ± 0.2	(5.1 ± 2.1)E14	2.25E-50
Abell 611	1.1 ± 0.1	(4.0 ^{+0.7} _{-0.8})E14	2.29E-50
Abell 621	1.2 ^{+0.2} _{-0.1}	(4.8 ^{+1.7} _{-1.8})E14	2.11E-50
Abell 773	1.1 ± 0.1	(3.6 ± 1.2)E14	2.06E-50
Abell 781	1.1 ± 0.1	(4.1 ± 0.8)E14	2.34E-50
Abell 990	0.9 ± 0.1	(2.0 ^{+0.4} _{-0.1})E14	2.09E-50
Abell 1413	1.1 ± 0.1	(4.0 ± 1.0)E14	2.29E-50
Abell 1423	0.9 ± 0.1	(2.2 ± 0.8)E14	2.30E-50
Abell 1758a	1.1 ± 0.1	(4.1 ^{+0.7} _{-0.8})E14	2.34E-50
Abell 1758b	1.1 ± 0.2	(4.4 ± 1.9)E14	2.52E-50
Abell 2009	1.2 ± 0.1	(4.6 ± 1.5)E14	2.03E-50
Abell 2111	1.1 ± 0.1	(4.2 ± 0.9)E14	2.40E-50
Abell 2146	1.2 ± 0.1	(5.0 ± 0.7)E14	2.20E-50
Abell 2218	1.3 ± 0.1	(6.1 ± 0.9)E14	2.11E-50
RXJ0142+2131	1.0 ± 0.1	(3.7 ^{+1.1} _{-1.2})E14	2.82E-50
RXJ1720+2638	0.9 ± 0.1	(2.0 ± 0.4) E14	2.09E-50
Average			2.27E-50
St. deviation			1.96E-51

Table 9 Error limit of Λ for CLASH survey

Cluster	Radius (Mpc)	$M_{vir} (M_{\odot})$	$\Lambda (m^{-2}) \leq$
Abell 383	1.86	(1.04 ± 0.07)E15	3.31E-51
Abell 209	1.95	(1.17 ± 0.07)E15	2.87E-51
Abell 2261	2.26	(1.76 ± 0.18)E15	4.74E-51
RXJ2129+0005	1.65	(0.73 ± 0.07)E15	4.74E-51
Abell 611	1.79	(1.03 ± 0.07)E15	3.71E-51
MS2137-2353	1.89	(1.26 ± 0.06)E15	2.70E-51
RXCJ2248-4431	1.92	(1.40 ± 0.12)E15	5.16E-51
MACSJ1115+0129	1.78	(1.13 ± 0.10)E15	5.39E-51
MACSJ1931-26	1.61	(0.83 ± 0.06)E15	4.37E-51
RXJ1532.8+3021	1.47	(0.64 ± 0.09)E15	8.62E-51
MACSJ1720+3536	1.61	(0.88 ± 0.08)E15	5.83E-51
MACSJ0429-02	1.65	(0.96 ± 0.14)E15	9.48E-51
MACSJ1206-08	1.63	(1.00 ± 0.11)E15	7.73E-51
MACSJ0329-02	1.54	(0.86 ± 0.11) E15	9.16E-51
RXJ1347-1145	1.80	(1.35 ± 0.19)E15	9.91E-51
MACSJ1311-03	1.28	(0.53 ± 0.04)E15	5.80E-51
MACSJ1423+24	1.34	(0.65 ± 0.11)E15	1.39E-50
MACSJ0744+39	1.33	(0.79 ± 0.04)E15	5.17E-51
CLJ1226+3332	1.57	(1.72 ± 0.11)E15	8.65E-51
Average			6.38E-51
SD			2.85E-51

galaxy systems, tracing galaxy pairs, groups and clusters. It is important that the used data are not of the same origin but were obtained at galactic surveys, gravity lensing studies and by Planck satellite. For the analysed hierarchy of systems of galaxies the numerical value of Λ obtained as a weak field GR without any cosmological considerations is in visible agreement with Λ obtained from relativistic cosmology.

There are at least two aspects to be outlined at the interpretation of the obtained Λ s regarding both the values and their scatter:

- (a) Equation (6) used for the estimation of Λ assumes that the virial theorem, i.e. the equipartition is determined purely due to the Λ term, while obviously the conventional Newtonian term's contribution has to be there as well. Therefore, the cosmological constant Eq. (5) has to be a lower limit for the Λ values obtained for galaxy systems. Namely, for fully virialized systems the empirical values of Λ have to be close to the cosmological constant, Eq. (5), while for others the empirical values have to be higher. Exactly such a behavior is visible in the exhibited Tables;
- (b) the scatter in values of Λ s in Tables is certainly expected due to the various virialization degree reached in each given system (galaxy pair, group, cluster). Decrease of that scatter will be possible when more refined dynamical structure of any individual system can be available.

Thus, while for certain systems e.g. galaxy groups of NGC 2894, NGC 3049, NGC 3810, P 51971 the obtained Λ s are close to the cosmological constant value, Eq. (5), in their absolute majority they exceed the latter.

The obtained values of Λ are also influenced by the inhomogeneity of the data sources, e.g. by the inevitable difference between the lensing and dynamical masses of galaxy systems or the masses of CMB-SZ clusters (see [18]). The structure and the extension of the dark halos of galaxies [19] are especially relevant for pair galaxies and galaxy groups studies. Note that, a crucial difference of the modified potential Eq. (2) from the Newtonian one is that, the second term in Eq. (2) defines non-force free field inside a shell [20] and hence e.g. the properties of galactic disks have to be determined by dark halos [1, 19]. The Λ -constant can be associated also to the time arrow [21]. Regarding the Λ -constant, including in the context of the local Universe within equilibria concepts, see [22–25]. Among other means for probing weak-field modified gravity are the large scale matter distribution (see [26]), the effects in Solar system [27, 28].

Thus, the Λ -constant enables the description both of the accelerated universe and the dynamics of galactic systems. The cosmology is described by GR, while the galaxy system dynamics is described by weak field GR, both containing the same Λ . One can therefore conclude that, the non-zero cos-

mological constant might be discovered from galactic systems even before high redshift SN surveys and CMB.

Acknowledgements We are thankful to the referee for valuable comments. AS acknowledges the receipt of the grant AF-04 from the Abdus Salam International Centre for Theoretical Physics, Trieste, Italy.

Data Availability Statement This manuscript has no associated data or the data will not be deposited. [Authors' comment: This manuscript has no associated data or the data will not be deposited because only published data are used.]

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