

**Dr. Arsen Narimanyan**
Bibliothekstraße 1
Department of Mathematics and
Computer Science
28359 Bremen
GermanyTelefon +49 (0)421 218 - 63570
Mobile +49 (0)177 181 - 83 93
arsen@uni-bremen.de

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Review of Thesis
on
**“Some Nonlinear Integral Equations
in the Theory of p-adic Strings”**
for attaining a scientific degree of
Candidate of Phys-Mathematical Sciences

Verwaltung/Sekretariat
Kerstin Düren
Telefon +49 (0421) 218 - 63720

submitted by
Metaksya Avetisyan

String theory addresses one of the deepest problems of contemporary theoretical physics, namely the reconciliation of gravity and quantum theory. It studies the interaction of so called quantum strings by applying the ideas of both quantum mechanics and the relativity theory. The main aim of the string theory is to create a unified field theory. String theory has also fostered a fascinating interplay between mathematics and physics.

The starting point in the field of p-adic string theory is due to V. S. Vladimirov going back to 1980s and dealing with models describing the change of non-Archimedean geometry of space on Planck scales. The investigation of such p-adic models leads to the study of nonlinear integral equations, which are defined on unbounded domains. The crucial point is that the operators in the equation are non-compact and critical. Critical means that, besides the so called vacuum (trivial) solutions, the equations may have non-trivial physical solutions. The main difficulty for the construction of non-trivial solutions is that the classical fixed point principles do not work for such integral equations which significantly complicates the investigation of issues on solvability of equations.

The present work of Metaksya Avetisyan is a contribution in the field of string theory and is devoted to the issues of solvability of some nonlinear integral equations and their discrete analogues arising in p-adic string theory as well as in the theory of spatial-temporal spread of epidemics.

Problems and Obtained Results

The study of Metaksya Avetisyan is very broad. In her work she considered three mathematical problems:

1. System of convolution type integral equations with cubic nonlinearity.

Imposing certain natural conditions on kernel functions, the author proved in the first chapter the existence of a non-trivial solution of the system. Moreover, the solution has been obtained by construction and shown to be continuous, monotone, non-decreasing and bounded. A two-sided estimation for the solution of the system has been obtained as well (see Theorem 1.1).

Another interesting result here is the proof of the existence of one parameter family of solutions and its asymptotic behaviour. Mrs. Avetisyan showed that the difference of the solution and its limit belongs to the class of integrable functions (Theorem 1.2 and 1.3).

2. Some nonlinear multidimensional integral equations of convolution type.

Assuming in Chapter 2 that the function describing the nonlinearity is odd, continuous on \mathbf{R} and satisfies three natural conditions listed on page 39, the author proved the existence of a non-trivial, bounded and in each argument monotonously non-decreasing solution. A two-sided estimation for the solution has been obtained here as well.

Moreover, it has been shown that the integral equation possesses a n -parametric family of nontrivial solutions, which are continuous and bounded.

3. Nonlinear infinite algebraic system of equations with Teoplitz-Hankel matrix.

The Chapter 3 is devoted to the discrete analogue of considered integral equations. These equations arise in the modelling of spatial-temporal spread of epidemics, which is a new and essential field of investigation for mathematicians. The study of such discrete equations is not a trivial task and includes several theoretical difficulties. Therefore there are no advanced research results in this area nowadays.

Nevertheless, Mrs. Avetisyan took the risk and obtained basic results on the existence and uniqueness of the solution in the space of bounded sequences (Theorem 3.1). Additionally, she discussed the asymptotic behaviour of the solution and formulated the results in Theorems 3.2-3.5.

Evaluation

The work of Metaksya Avetisyan is done well and without any mistake. All proofs are done in detail and with high accuracy. The results obtained by Mrs. Avetisyan generalize a number of known theorems. They are very interesting and can be applied in different areas of mathematical physics and epidemics. Some practical examples are discussed by the author in the work as well. As a reviewer of the thesis, I have been mostly admired from the fact, that the existence of solutions of complex nonlinear equations have been done constructively. This provides a nice basis for numerical calculations.

Some critical remarks in favour of Mrs. Avetisyan's work can be made as well. As already mentioned, the constructive approach for solutions allows to illustrate the solutions numerically. The equations are highly applicable in real life and it would be very interesting to see how the solutions look like, at least for small dimensions.

A small section with open problems and further discussions might be helpful, as one could be then able to see how the author handles the possible extensions of the present work.

Nevertheless, the study of Mrs. Metaksya Avetisyan is a well done work. The equations discussed in the work are highly nonlinear, the investigation of which contain a solid background of several mathematical theories. She has made it to conduct a very interesting research and deliver excellent mathematical results in the field of integral equations. All this together demonstrates the high scientific level of the work of Metaksya Avetisyan.

Summary

In general, the content of Mrs. Avetisyan's thesis and its demonstration gives a very good impression. A huge amount of scientific work has been done, which can be considered to be a serious and important contribution in the field of differential equations and mathematical physics.

Hence, without any doubt, I endorse that the thesis of Metaksya Avetisyan fulfils the requirements of the Qualifying Committee of the Republic of Armenia, and Mrs. Avetisyan deserves to obtain the scientific degree of

Candidate of Phys-Mathematical Sciences

in the field of Differential Equations and Mathematical Physics (A.01.02).

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Dr. rer. nat. Arsen Narimanyan
Department of Mathematics and Computer Science
University of Bremen
Bremen, Germany

