ՀՀ ԿՐԹՈՒԹՅԱՆ, ԳԻՏՈՒԹՅԱՆ, ՄՇԱԿՈՒՅԹԻ ԵՎ ՍՊՈՐՏԻ ՆԱԽԱՐԱՐՈՒԹՅՈՒՆ
ԵՐԵՎԱՆԻ ՊԵՏԱԿԱՆ ՀԱՄԱԼՍԱՐԱՆ

ՍԱՐԳՍՅԱՆ ՀԱՅԿ ԳՈՒՐԳԵՆԻ

# ԴԱՇՏԻ ՔՎԱՆՏԱՅԻՆ ՏԵՍՈՒԹՅԱՆ ՈՐՈՇ ԵՐԵՎՈՒՅԹՆԵՐ ԱՐՏԱՔԻՆ ԳՐԱՎԻՏԱՑԻՈՆ ԴԱՇՏՈՒՄ

Ա.04.02 - <<Տեսական ֆիզիկա>> մասնագիտությամբ
ֆիզիկամաթեմատիկական գիտությունների թեկնածուի
գիտական աստիճանի հայցման ատենախոսության

ՍԵՂՄԱԳԻՐ

ԵՐԵՎԱՆ - 2020

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MINISTRY OF EDUCATION, SCIENCE, CULTURE AND SPORT OF RA
YEREVAN STATE UNIVERSITY

SARGSYAN HAYK

# SOME EFFECTS OF QUANTUM FIELD THEORY IN EXTERNAL GRAVITATIONAL FIELD

Thesis for the degree of Candidate of physical and mathematical sciences
Speciality 01.04.02 - "Theoretical Physics"

ABSTRACT

YEREVAN - 2020

Ատենախոսության թեման հաստատվել է Երևանի պետական համալսարանում

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| Գիտական ղեկավար՝ | ֆիզ.-մաթ. գիտ. դոկտոր, պրոֆեսոր Արամ Ազատի Սահարյան |
| Պաշտոնական ընդդիմախոսներ՝ | ֆիզ.-մաթ. գիտ. դոկտոր, պրոֆեսոր Գևորգ Սուրենի Հաջյան<br>ֆիզ.-մաթ. գիտ. դոկտոր Ռուբիկ Հրաչիկի Պողոսյան |
| Առաջատար կազմակերպություն՝ | ՀՀ ԳԱԱ Ֆիզիկայի կիրառական պրոբլեմների ինստիտուտ |

Ատենախոսության պաշտպանությունը կայանալու է 2020թ. դեկտեմբերի 19-ին ժամը 12:00-ին Երևանի պետական համալսարանում գործող Ֆիզիկայի 049 Մասնագիտական խորհրդի նիստում:

Հասցե՝ 0025 Երևան, Ալեք Մանուկյան փ. 1, ԵՊՀ

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| Մասնագիտական խորհրդի գիտական քարտուղար՝ |  | ֆիզ.-մաթ. գիտ. թեկնածու Վ.Պ. Քալանթարյան |

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The thesis theme is approved at the Yerevan State University

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| Scientific supervisor: | Doctor of Phys. Math. Sciences Prof. Aram Saharian |
| Official opponents: | Doctor of Phys. Math. Sciences, Prof. Gevorg Hajyan<br>Doctor of Phys. Math. Sciences Rubik Poghossian |
| Leading organization: | Institute of Applied Problems of Physics of NAS RA |

The defense of the thesis will take place at 12:00 on 19 December 2020 on the session of the Specialized Council 049 Physics of the Yerevan State University.

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The thesis is available in the library of the Yerevan State University.

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## GENERAL DESCRIPTION OF THE WORK

**Relevance of topic.** Despite the tremendous efforts of physicists, the quantum theory of gravity has not yet been constructed. The studies of the influence of the gravitational field on quantum effects are carried out within the framework of semiclassical theory where the gravity is considered as a classical geometry of background spacetime and the other fields are described by quantum field theory. This approach has wide range of applications and a number of interesting effects have been predicted. In particular, they include the polarization of vacuum by the gravitational field, the quantum evaporation of black holes and particle creation in the early stages of the Universe expansion. In currently the most popular model, the quantum fluctuations of fields during the inflationary epoch serve as seeds for the large scale structure formation in the Universe. This model is in good agreement with the observational data for the temperature anisotropies of the cosmic microwave background radiation.

Among the most interesting directions in quantum field theory on curved spacetime is the investigation of the vacuum polarization by the gravitational field. The exact results for the vacuum characteristics are obtained for highly symmetric background geometries only. In particular, the de Sitter and anti-de Sitter (AdS) geometries have attracted a great deal of attention. Along with the Minkowski spacetime, these manifolds are maximally symmetric solutions of the Einstein equations with positive and negative cosmological constants in the right-hand side and a large number of problems are exactly solvable on their background. In addition to high symmetry, the importance of the AdS spacetime in quantum field theory is motivated by a number of other reasons. The AdS spacetime is not globally hyperbolic and the early interest was mainly related to principal questions of the quantization procedure in such spacetimes. To yield well defined dynamics, boundary conditions should be chosen on timelike conformal infinity and this brings several qualitatively new features compared to the Minkowskian theories. In particular, new type of instabilities may arise. The importance of such studies is also due to the appearance of AdS spacetime as a ground state in extended supergravity and in string theories and as near horizon limit of extremal black holes and black strings. The non-zero curvature of the AdS spacetime provides an infrared regulator for correlation functions, consistent with supersymmetry and modular invariance [1]. Another new feature of the dynamics, that distinguishes the AdS bulk from the Minkowski one, is the existence of consistent theories for interacting higher spin fields.

The geometrical properties of the AdS spacetime play a crucial role in two fascinating modern developments of high-energy physics. The first one, the AdS/CFT correspondence [2], is a realization of the holographic principle. It states a duality between theories formulated in different numbers of spacetime dimensions: the supergravity or string theory in AdS bulk and conformal field theory on its boundary. Among the most important implications of this correspondence is the possibility for the investigation of nonperturbative effects in one theory through the weak coupling expansion of the dual theory. In addition to the high-energy physics, the recent developments include applications in condensed matter physics (holographic superconductors, quantum phase transitions, and topological insulators) [3]. The second focus of intense interest with AdS spacetime as the background geometry is various types of braneworld models [4] where the standard model fields are restricted to a hypersurface (brane) embedded in a higher dimensional spacetime. Initially proposed for a resolution of the gauge hierarchy problem, these models give new insights into various problems of particle physics and cosmology. The existence of the branes on which the matter fields are confined is predicted also by string theories.

**The aim of the thesis** is to investigate the properties of the quantum vacuum for scalar, Dirac and electromagnetic fields in background geometries with curvature, with nontrivial topology and in the presence of boundaries. In some problems simple configurations of an external classical gauge field is assumed. All the problems are exactly solvable and closed analytical expressions for physical characteristics of the vacuum state are provided. We have considered:

- The vacuum expectation value (VEV) of the surface energy-momentum tensor on branes in locally AdS spacetime with compact spatial dimensions, induced by quantum fluctuations of a bulk charged scalar field with general curvature coupling parameter.
- The vacuum currents for a charged Dirac field in the geometry of two parallel branes in background of locally AdS spacetime with a part of spatial dimensions compactified on a torus.
- The electromagnetic field vacuum correlators, the VEVs of the local characteristics of the electromagnetic vacuum and the Casimir forces acting on two plates in AdS spacetime, parallel to the AdS boundary
- The ground state fermionic currents in finite length two-dimensional conical rings for different boundary conditions on the ring edges and in the presence of a magnetic flux threading the ring.

**Scientific novelty.** The vacuum fluctuations of a bulk quantum scalar field induce a gravitational source on branes in AdS spacetime that is of cosmological constant type. The corresponding VEV for the surface energy-momentum tensor is investigated for the general case of spatial dimension and of the number of compat dimensions. It is decomposed into single brane and the second brane-induced contributions. The divergences in the single brane part are renormalized by the zeta function method. It is shown that in the generalized Randall-Sundrum model with two branes the cosmological constant generated on the visible brane is exponentially suppressed for large interbrane separations. For charged quantum fields an important characteristic of the vacuum state is the expectation value of the current density. In models with compact dimensions the vacuum currents are induced by nontrivial pases in the periodicity conditions along compact dimensions or by gauge field fluxes enclosed by those dimensions. In the geometry with two parallel branes on locally AdS spacetime with toral dimensions, closed analytic expressions are obtained for the VEV of the fermionic current density. The behavior of the latter is investigated as a function of the interbrane distance, of the lengths of compact dimensions, of the magnetic flux and of the boundary conditions imposed on the fermionic fields at the locations of the branes. In particular, the latter include the boundary conditions for the fields odd and even under the reflections with respect to the branes in $Z_2$-symmetric braneworld models. It is shown that the influence of the background gravitational field on the vacuum currents is essential at separations between the branes larger than the curvature radius of the spacetime. The complete set of the electromagnetic modes is determined for two parallel plates in AdS spacetime and for two types of boundary conditions generalizing the perfect conducting boundary conditions in QED and confining boundary conditions for gluons in QCD. Exact expressions are provided for the vacuum correlators of the vector potential and for the field tensor. On the base of them the VEVs of the photon condensate, of the electric and magnetic fields squared, and of the energy-momentum tensor are investigated. All

them are presented in the form where the boundary-free, single plate and the second plate contributions are explicitly separated. The Casimir forces acting on the plates are investigated. They are decomposed into the self-action and the interaction parts. The first one needs and additional renormalization that can be done by the generalized zeta function method. The interaction forces are attractive for all separations between the branes. As a simplified model for a cosmic string we consider a two-dimensional conical ring. The combined influence of the planar angle deficit, of the edges and of the magnetic flux threading the ring on the ground state expectation values for charge and current densities is investigated for a Dirac field. Different boundary conditions are discussed on the ring edges and the behavior of the VEVs in asymptotic regions of the parameters is discussed. Applications are given for graphene conical rings the long wavelength excitations of which are described by the Dirac model.

**Practical importance.** The vacuum fluctuations of bulk fields induce a surface energy-momentum tensor on the branes that may serve as a mechanism for the generation of cosmological constant from the viewpoint of an observer living on the brane. For the locally AdS background geometry and for separations between the branes larger than the curvature radius, the cosmological constant is strongly suppressed and can serve as a model for the dark energy driving the accelerated expansion of the observable universe in the recent epoch. The scheme considered in the thesis can be generalized for more general geometries of the compact subspace. The complete sets of the fermionic and electromagnetic modes in the AdS bulk serve as a base for construction of the quantum electrodynamics in AdS spacetime with an arbitrary number of spatial dimensions. The fermionic modes can also be used for the investigation of the VEVs for the energy-momentum tensor and for fermionic condensate. The combined effect of the fermionic and electromagnetic Casimir forces acting on the branes may provide a stabilization mechanism for stabilization of the interbrane distance. The latter is required to prevent the variations of physical constants in braneworlds. The results for the fermionic charge and current densities can be used for the investigation of the influence of the curvature, topology and edges on the persistent currents in curved graphene tubes and in conical graphene rings. Curved graphene structures provide an important laboratory for the investigation of curvature and topological effects in quantum field theory. The special case $D = 2$ of the model considered in the thesis presents an exactly solvable problem of that kind.

**Basic results to be defended:**

1. The vacuum fluctuations of a bulk complex scalar field generate a gravitational source of the cosmological constant type on branes in locally AdS spacetime with a part of spatial dimensions compactified to a torus. The induced cosmological constant is an even function of the magnetic flux enclosed by compact dimensions with the period equal to the flux quantum. Depending on the boundary conditions on the branes and on the magnetic flux, it can be either positive or negative. In the generalized Randall-Sundrum barneworld model with two branes and for separations solving the hierarchy problem between the Planck and electroweak energy scales, the cosmological constant on the visible brane is exponentially suppressed and can serve as a model for dark energy driving the accelerated expansion of the Universe at recent epoch.

2. In the geometry of two branes on background of locally AdS spacetime with toroidally compact dimensions, the enclosed magnetic flux and nontrivial phases in the quasiperiodicity conditions along those dimensions induce vacuum fermionic currents. The contributions of the branes in the corresponding expectation values are separated explicitly and their behavior near the AdS boundary and horizon is investigated. The

influence of the gravitational field on the current density is essential for separations between the branes larger than the AdS curvature radius. Depending on the boundary conditions for the fermionic field at the locations of the branes, the latter can either increase or decrease the vacuum currents. In braneworld models the current density along compact dimension is a source of magnetic fields on the visible brane. In the special case of two spatial dimensions with one of them compactified to a circle, the results are applied for investigation of edge effects on the current density in curved graphene tubes.

3. Closed analytic expressions are obtained for the correlators of the electromagnetic field vacuum fluctuations in the region between two plates in AdS spacetime. The vacuum expectation values of the electric and magnetic fields squared and for the energy-momentum tensor are investigated. The interaction parts in the Casimir forces are attractive for both conducting and confining boundary conditions on the plates. At separations between the plates larger than the AdS curvature scale the gravity essentially changes the behavior of the Casimir forces: they decay exponentially in contrast to the power-law decay for the Minkowski bulk. Another feature differing the AdS and Minkowski bulks is that the forces on the left and right plates differ. The results are applied for vector fields in braneworld models with even and odd parities.

4. The effects of planar angle deficit and boundaries on the VEVs of the charge and current densities are studied for a fermionic field in (2+1)-dimensional spacetime. The VEVs are decomposed into the boundary-free and edge-induced contributions. Both of them are periodic functions of the magnetic flux treading the conical ring and increase by the absolute value with increasing planar angle deficit. On the edges the azimuthal current density is equal to the charge density or has an opposite sign. The charge and current densities are investigated in parity and time-reversal symmetric fermionic models. If the boundary conditions and the phases in quasiperiodicity conditions for separate fields, realizing the irreducible representations of the Clifford algebra, are the same the total charge density vanishes. Applications are given to finite length graphitic cones.

**Approbation of the work.** The results of the thesis were reported at the conferences "Cosmology and the Quantum Vacuum" (Benasque, Spain, 2016), "Instability phenomena and evolution of the Universe" (Byurakan, Armenia, 2018), "Modern Physics of Compact Stars and Relativistic Gravity" (Yerevan, Armenia, 2019) and have been discussed at the seminars of the Chair of Theoretical Physics of Yerevan State University and of the INFN National Laboratory of Frascati (Frascati, Italy).

**Publications.** Five papers are published in the topic of the thesis.

**Structure of the thesis**. The thesis consists of Introduction, four Chapters, Summary and the list of references. It contains 176 pages, including 43 figures.

## CONTENT OF THE THESIS

In **Introduction** the scientific literature related to the topic of the thesis is reviewed, the relevance of the topic is argued, the aim of the work, the scientific novelty and the practical value are presented, the basic results to be defended are described.

In **Chapter 1** the VEV of the surface energy-momentum tensor for a charged scalar field $\varphi(x)$ is investigated in $(D+1)$-dimensional locally AdS spacetime with compact spatial dimensions

in the geometry of branes parallel to the AdS boundary. In Poincaŕe coordinates the metric tensor is given by the line element

$$ds^2 = g_{ik}dx^i dx^k = (a/z)^2 \left(\eta_{\mu\nu}dx^\mu dx^\nu - dz^2\right), \tag{1}$$

where $a$ is the curvature radius, $\eta_{\mu\nu} = \mathrm{diag}(1, -1, \ldots, -1)$ is the $D$-dimensional Minkowski metric, $\mu, \nu = 0, 1, \ldots, D-1$. The coordinate $z$ spans the range $0 < z < \infty$ and the hypersurfaces $z = 0$ and $z = \infty$ correspond to the AdS boundary and horizon respectively. The notion of vacuum in quantum field theory has a global nature and its properties are sensitive to both the local and global characteristics of the background spacetime. In the problems considered in chapter 1, the local geometry of the background spacetime is that of an AdS, given by (1), however, the global properties differ. Namely, we consider a model where the subspace normal to the coordinate $z$ has the topology $R^p \times T^q$, with $p$ and $q$ being positive integers such that $p + q = D - 1$, and $T^q$ denotes a $q$-dimensional torus. The ranges of the coordinates are given by $-\infty < x^i < +\infty$ for $i = 1, 2, \ldots, p$, and by $0 \leq x^i \leq L_i$ for $i = p+1, \ldots, D-1$, where $L_i$ is the coordinate length of $i$-th compact dimension.

The field equation reads

$$\left(g^{ik}D_i D_k + m^2 + \xi R\right)\varphi(x) = 0, \tag{2}$$

where $D_k = \nabla_k + ieA_k$ is the covariant derivative operator that includes the part related to the presence of a gauge field $A_k$, $R$ is the scalar curvature and $\xi$ is the curvature coupling parameter. Along the compact dimensions the field obeys quasiperiodicity conditions with constant phases $\alpha_l$:

$$\varphi(t, x^1, \ldots, x^l + L_l, \ldots, y) = e^{i\alpha_l}\varphi(t, x^1, \ldots, x^l, \ldots, y), \quad l = p+1, \ldots, D-1. \tag{3}$$

A simple configuration of constant gauge field $A_k$ is assumed. Although by a gauge transformation one can pass to a new gauge where the vector potential is zero, $A_k$ will enter in the quasiperiodicity conditions for the new scalar field by changing the phases $\alpha_l$ to $\tilde{\alpha}_l = \alpha_l + eA_l L_l$. The term $eA_l L_l$ is formally interpreted in terms of the magnetic flux enclosed by the $l$th compact dimension. As a consequence, an influence of Aharonov-Bohm type arises on the physical properties of the vacuum state which is related to the nontrivial topology of the background geometry. As a boundary geometry we assume the presence of co-dimension one branes located at $z = z_1$ and $z = z_2$, $z_1 < z_2$. On the branes the field operator obeys Robin boundary conditions

$$(1 + \beta^{(j)} n^k_{(j)} D_k)\varphi(x) = 0, \ z = z_j, \tag{4}$$

where $\beta^{(j)}$, $j = 1, 2$, are real constants and $n^i_{(j)}$ denotes the inward pointing normal to the brane at $z = z_j$.

The branes divide the space into three regions: region between the brane $z = z_1$ and AdS boundary (L-region), region between two branes and region between the brane at $z = z_2$ and AdS horizon (R-region). In the first and third regions the VEV of the surface energy-momentum tensor coincides with that for the geometry of a single brane with appropriate location and we first consider that geometry. Though the background space is homogeneous the extrinsic curvature tensor for the brane is different from zero and for that reason the properties of the vacuum in the L- and R-regions are different. The corresponding VEVs are discussed separately. The surface energy-momentum tensor is located on the brane and its VEV contains surface divergences well-known from quantum field theory on manifolds with boundaries. For regularization of those divergences and further renormalization we use the generalized zeta

function technique. The pole and finite parts in the VEV of the surface energy-momentum tensor are explicitly separated. In general, the finite part depends on the renormalization mass scale. The pole parts cancel out in the total surface energy density for odd values of the spatial dimension if the coefficients in the boundary condition for the L- and R-regions have the same absolute value but different signs. In this special case the dependence of the finite part on the renormalization mass scale disappears. The VEV of the energy density induced by quantum effects of the bulk field $\varphi(x)$ and the corresponding pressure are related by the equation of state $p = -\varepsilon$, which corresponds to a cosmological constant on the brane. Depending on the parameters of the problem, the induced cosmological constant can be either positive or negative. It is an even periodic function of the magnetic flux enclosed by compact dimensions with the period equal to the flux quantum.

In the region between two branes the surface energy densities on separate branes are decomposed into two contributions. The first one corresponds to the energy density on a single brane when the second brane is absent and the other contribution is induced by the presence of the second brane. Unlike the single brane part, the second brane-induced contribution does not require an additional renormalization. A closed analytic expressions are provided for it that contain strongly convergent integrals and are well adapted for asymptotic and numerical analysis. Various limiting cases are discussed in detail. In particular, it is shown that at large separations between the branes the second brane-induced contributions to the surface energy density are exponentially suppressed and give rise to exponentially small cosmological constant. For large values of the length $L_l$ of the $l$th compact dimension the leading order term in the corresponding asymptotic expansion coincides with the surface energy density in the geometry where the $l$th dimension is decompactified. For small values of $L_l$ compared to the other length scales, the behavior of the surface energy density is essentially different for the cases $\tilde{\alpha}_l = 0$ and $\tilde{\alpha}_l \neq 0$. In the first case there is a zero mode and the dominant contribution comes from it. The corresponding VEV is expressed in terms of the surface energy density in the $D$-dimensional model where the $l$th dimension is excluded. In the case $\tilde{\alpha} \neq 0$ and for small values of $L_l$ the second brane-induced surface energy density is suppressed by the factor $e^{-2|\tilde{\alpha}_l| z_2/L_l}$.

The ratio of the induced cosmological constant on the brane to the corresponding Planck scale quantity is considered for separate branes. Among the main motivations of the Randall-Sundrum type braneworld models on the AdS bulk is the possibility for the generation of large hierarchy between the Planck and electroweak energy scales for moderate values of the distance between the branes in units of the AdS curvature radius. For those values of the interbrane distance, we have estimated the ratio of the cosmological constant induced by the hidden brane on the visible brane to the Planck energy scale on the visible brane. It is shown that in $D = 5$ model, for separations of the branes solving the hierarchy problem, the energy density induced by the hidden brane on the visible brane is of the order of the dark energy required to explain the accelerated expansion of the Universe at recent epoch.

In **Chapter 2** the effects of two parallel branes on the VEV of the current density $j^\mu = e\bar{\psi}\gamma^\mu\psi$ for a charged fermionic field $\psi(x)$ are investigated in locally AdS spacetime with a part of spatial dimensions compactified to a torus. The corresponding investigations in the absence of branes and in the geometry with a single brane are presented in [5,6]. The local geometry and the topology of the background spacetime are the same as in Chapter 1. On the branes the field obeys the boundary conditions

$$(1 \pm i\gamma^k n_{(j)k})\psi(x) = 0, \ z = z_j, \tag{5}$$

where $\gamma^k$ are the curved space Dirac matrices being $N \times N$ matrices with $N = 2^{[(D+1)/2]}$ (the

square brackets mean the integer part). The conditions with the upper sign correspond to the boundary condition used in MIT bag models of the hadrons to confine the quarks. The quasiperiodicity conditions for the field $\psi(x)$ along compact dimensions are similar to (3), with the phases $\alpha_l$, and the presence of a constant gauge field is assumed. The complete set of the fermionic mode functions in the region between the branes is found. The $z$-dependence of the modes is expressed in terms of the cylindrical functions. The eigenvalues of the corresponding quantum number are the zeros of combinations of the Bessel and Neumann functions. The mode sum for the VEV of the current density contains series over those eigenvalues. In order to find an integral representation, convenient in asymptotic analysis and numerical calculations, we have used a variant of the generalized Abel-Plana formula that allowed to extract explicitly the brane-induced contributions. For all the boundary conditions discussed, the VEVs of the charge density and of the components of the current density along non-compact dimensions vanish.

In the investigation of the current density along compact dimensions, first we consider the case of the bag boundary condition on the branes which is the most frequently used one for confinement of fermionic fields. In the region between the branes the $l$th component of the current density is presented as

$$\begin{aligned}\langle j^l \rangle &= \langle j^l \rangle_0 + \frac{(4\pi)^{-(p+1)/2} N e z^{D+2}}{\Gamma\left((p+1)/2\right) V_q a^{D+1}} \sum_{\mathbf{n}_q} k_l \int_{k_{(q)}}^{\infty} du\, u (u^2 - k_{(q)}^2)^{\frac{p-1}{2}} \\ &\times \left[\frac{K_{ma+1/2}(z_1 u) I_{ma-1/2}(z_2 u)}{I_{ma+1/2}(z_1 u) K_{ma-1/2}(z_2 u)} + 1\right]^{-1} \sum_{j=\pm 1} j \Big[2j K_{ma+j/2}(zu) I_{ma+j/2}(zu) \\ &+ \frac{I_{ma-1/2}(z_2 u)}{K_{ma-1/2}(z_2 u)} K_{ma+j/2}^2(zu) - \frac{K_{ma+1/2}(z_1 u)}{I_{ma+1/2}(z_1 u)} I_{ma+j/2}^2(zu)\Big], \end{aligned} \tag{6}$$

where $e$ and $m$ are the charge and the mass of the field, $V_q = L_{p+1} \times \cdots \times L_{D-1}$ is the volume of the compact subspace, $I_\nu(x)$ and $K_\nu(x)$ are the modified Bessel functions, and $k_{(q)}^2 = \sum_{i=p+1}^{D-1} k_i^2$ is the momentum squared in the compact subspace. The component of the momentum along the $i$th compact dimension, $k_i = (2\pi n_i + \tilde{\alpha}_i)/L_i$, $n_i = 0, \pm 1, \pm 2, \ldots$, is quantized by the corresponding quasiperiodicity condition. In (6), the summation goes over the set $\mathbf{n}_q = (n_{p+1}, \ldots, n_{D-1})$. The term $\langle j^l \rangle_0$ on the right hand side of the equation (6) is the current density in the problem where the background geometry is the same but the branes are absent. The equation (6) shows that the current density can be written as a sum of two contributions, $\langle j^l \rangle = \langle j^l \rangle_0 + \langle j^l \rangle_b$, where $\langle j^l \rangle_b$ is induced by the branes. The vacuum currents in the absence of the branes have been investigated in earlier works and here we are mainly concerned about the brane-induced effects. We have also provided representations with the separated contribution of the second brane when one adds it to the configuration with a single brane. The effects of the phases in the quasi-periodicity conditions and of the gauge field are encoded in the parameters $\tilde{\alpha}_i$ as in the previous chapter. The $l$th component of the current density is an odd periodic function of $\tilde{\alpha}_l$ and even periodic function of $\tilde{\alpha}_i$, $i \neq l$, with the period $2\pi$. In terms of the magnetic flux enclosed by the compact dimensions, this corresponds to the periodicity with the period equal to the flux quantum. To investigate the near-brane asymptotic we have also derived an alternative representation of the current density which contains an exponentially convergent sum over the eigenvalues of the radial quantum number. The new representation also explicitly shows the finiteness of the current density on the branes. The latter feature is in clear contrast to the on-brane behavior of the fermion condensate and of the VEV of the energy-momentum

tensor having surface divergences. The current density, integrated over the region between the branes, is connected to the on-brane values of the current density by a simple relation $\int_{z_1}^{z_2} dz\, \sqrt{|g|}\langle j^l\rangle = \left[z^{-D}a^{D+1}\langle j^l\rangle\right]_{z=z_1}^{z=z_2}$, where $g$ is the determinant of the metric tensor.

The general expression for the current density is rather complicated and, in order to clarify its behavior as a function of the parameters, we have considered various asymptotic limits. First of all, in the limit of large curvature radius the result is obtained for the geometry of two parallel plates in a locally Minkowski spacetime with a toroidal subspace. For a massless fermionic field, the problem under consideration is conformally related to the corresponding problem in locally Minkowski bulk and the current densities in these two geometries are connected by the standard formula. In the limit when the right brane tends to the AdS horizon, for fixed location of the left brane and of the observation point, the corresponding contribution to the current density is exponentially suppressed by the factor $e^{-2z_2k_{(q)}^{(0)}}$, where $k_{(q)}^{(0)2} = \sum_{i=p+1}^{D-1}\tilde{\alpha}_i^2/L_i^2$. When the location of the left brane tends to the AdS boundary, the corresponding contribution to the vacuum current decays like $z_1^{2ma+1}$.

It is also of interest to consider the behavior of the current in asymptotic regions of the length of compact dimensions. If the length of the $l$th dimension $L_l$ is much smaller than the other length scales in the problem, including the difference $z_2 - z_1$, the brane-induced contribution to the current density along that direction is suppressed by the factor $\exp[-2(z_2 - z_1)|\tilde{\alpha}_l|/L_l]$ and the total current is dominated by the brane-free part. For large values of $L_l$, the current density is dominated by the mode with the lowest value $\lambda_1$ of the radial quantum number and the current density is suppressed by the factor $\exp[-L_l\sqrt{\lambda_1^2/z_1^2 + k_{(q)}^{(0)2} - \tilde{\alpha}_l^2/L_l^2}]$. The behavior of the $l$th component of the current density for small values of the length $L_i$, $i \neq l$, crucially depends whether the phase $\tilde{\alpha}_i$, $|\tilde{\alpha}_i| < \pi$, is zero or not. For $\tilde{\alpha}_i = 0$ the dominant contribution comes from the zero mode along the $i$th dimensions and, to the leading order, the current density $\langle j^l\rangle$ is expressed in terms of the corresponding current density in $D$-dimensional spacetime with excluded $i$th dimension. In the case $\tilde{\alpha}_i \neq 0$, the VEV $\langle j^l\rangle$ is suppressed by the factor $e^{-2(z_2-z_1)|\tilde{\alpha}_i|/L_i}$.

The second class of boundary conditions considered in this chapter are the ones arising in $Z_2$-symmetric braneworld models and correspond to the lower sign in (5). The investigation of the vacuum currents in this case is done in a way similar to that in the case of the bag boundary condition. The corresponding current density in the region between the branes is also decomposed into boundary free and brane-induced contributions. For the mass range $ma < 1/2$, an important difference when compared to the bag boundary conditions appears in the limit when the left brane goes to the AdS boundary ($z_1 \to 0$). An additional contribution survives that can be interpreted as some kind of memory from the boundary condition we have imposed on the brane at $z = z_1$. Yet another two classes of boundary conditions arise in $Z_2$-symmetric braneworld models. Depending on the boundary conditions imposed, the presence of the branes can either increase or decrease the current density. In braneworld models of the Randall-Sundrum type the observers are localized on the visible brane and it is of interest to investigate the effects of the hidden brane on the current density on the visible brane. For this purpose, the part of the vacuum current induced by the hidden brane has been extracted explicitly. For the solution of the hierarchy problem between the electroweak and Planck energy scales it is required to have $z_2/z_1 \gg 1$. In this limit the behavior of the hidden brane-induced current essentially depends on the lengths of compact dimensions and is different for $z_1/L_i \gtrsim 1$ and $z_2/L_i \lesssim 1$.

In figure 1 we have depicted the brane-induced and total current densities along the compact

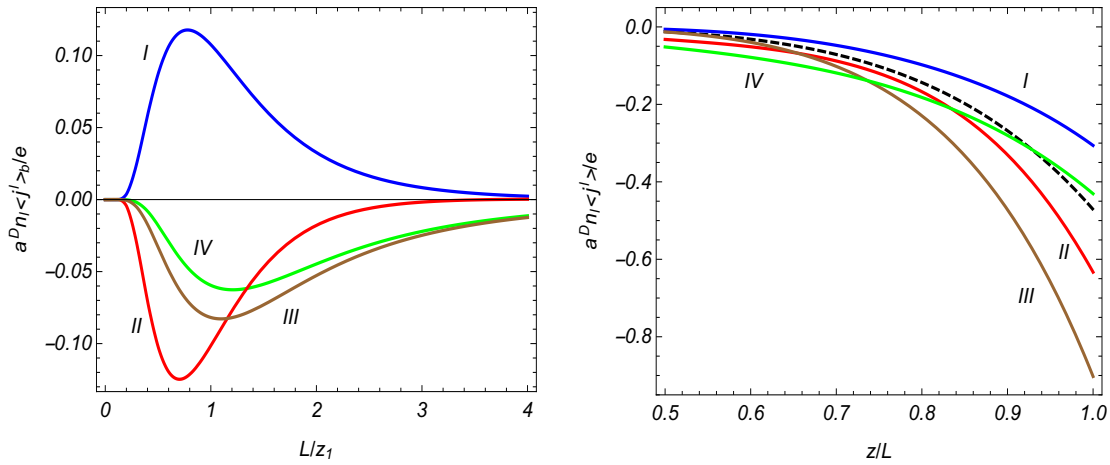


Figure 1: The brane-induced contribution in the current density (left panel) as a function of the length of the compact dimension in the case of $\tilde{\alpha} = \pi/2$, $ma = 1$, $z_2/z_1 = 2$, $z/z_1 = 1.5$. The total current density (right panel) in the region between the branes as a function of the coordinate $z$ in units of the length of the compact dimension. The dashed curve presents the current density in the absence of the branes. The Roman numbers near the curves correspond to different combinations of the boundary conditions (5) on separate branes.

dimension in the model with $q = 1$ as functions of the length of the compact dimension and the coordinate $z$, respectively. The plots are presented for four different boundary conditions: $(+,+)(I)$, $(-,-)(II)$, $(+,-)(III)$, and $(+,+)(IV)$. Here, the first sign in $(\cdot,\cdot)$ corresponds to the sign in the boundary condition (5) on the left brane and the second sign corresponds to the sign of the boundary condition on the right brane. The graphs are plotted for fixed values of the parameters given in the caption to the figure.

In odd-dimensional spacetimes, the models with massive fermionic fields realizing irreducible representations of the Clifford algebra are not parity and time-reversal invariant. Fermionic models with parity and time-reversal symmetry are constructed combining two fields corresponding to inequivalent representations. If the periodicity conditions along compact dimensions and the boundary conditions on the branes are the same for separate fields, when the current densities for those fields are the same as well and the expressions for the total current density is obtained from those presented with an additional factor two. However, both the periodicity and boundary conditions can be different for fields realizing inequivalent representations of the Clifford algebra. An example of $D = 2$ fermionic system with that type of situation is provided by semiconducting carbon nanotubes, with the electronic subsystem described by the Dirac model. In the corresponding setup the phases for separate fields have opposite signs and, in the absence of the magnetic flux, the corresponding current densities cancel each other if the boundary conditions for the fields are the same. In the case of different boundary conditions on the tube edges for separate fields, a nonzero current can be generated in the absence of magnetic flux. Curved graphene structures provide an important laboratory for the investigation of curvature and topological effects in quantum field theory. The special case $D = 2$ of our model presents an exactly solvable problem of that kind.

In **Chapter 3** we have discussed the effects of two parallel plates in AdS spacetime on the properties of the electromagnetic vacuum in an arbitrary number of spatial dimensions. The local geometry of the background spacetime considered in this chapter is the same as described in Chapter 1, however, no compact dimensions are assumed. As a boundary geometry two

plates parallel to the AdS boundary are considered. Two types of boundary conditions have been discussed on them. The first one is the generalization of the perfect conductor boundary condition for an arbitrary number of spatial dimensions and the second one corresponds to the confining boundary condition used in quantum chromodynamics to confine gluons. In the model under consideration the properties of the electromagnetic vacuum are encoded in two-point functions and we have started the consideration from the two-point function of the vector potential. It is presented in the form of the sum over complete set of electromagnetic modes. In the region between the plates, the eigenvalues of the quantum number corresponding to the direction normal to the plates are roots of an equation containing combinations of Bessel and Neumann functions. The application of a variant of the generalized Abel-Plana summation formula to the corresponding series allowed us to extract the contribution of the single plate and to present the second plate-induced part in the form that is well adapted for the evaluation of the VEVs for local observables. Two equivalent representations of the two-point function for the vector potential are provided. The VEVs of local physical observables are obtained from the two-point functions for the field tensor and we have derived the expressions for both the single plate and the second plate induced contributions.

As important local characteristics of the vacuum state, the VEVs of the electric and magnetic fields squared and the photon condensate are considered. The single plate and the second plate-induced contributions to the VEV of the electric field squared are positive for the perfect conductor boundary condition and negative for the confining boundary condition. The signs of the corresponding contributions in the VEV of the magnetic field squared and in the photon condensate are opposite to that for the electric field. Near the plates the VEVs are dominated by single plate contributions. In those regions the effects of the gravity on the VEVs are weak and the leading terms in the asymptotic expansions over the distance from the plate coincide with those in the corresponding problem on the Minkowski bulk. The effects of the gravity are essential for separations between the plates larger than the curvature radius of the background geometry. Having the correlators for the field tensor one can evaluate the Casimir-Polder forces acting on a polarizable particle. As an illustration we have considered the simplest case of isotropic polarizability in the static limit. Near the plate, the Casimir-Polder force is attractive (repulsive) with respect to that plate for the perfect conductor boundary condition (confining boundary condition).

Similar investigations for the VEV of the energy-momentum tensor are presented as well. The off-diagonal components vanish and the diagonal components are decomposed as the sum of the single plate contribution and the contribution induced by the presence of the second plate. The vacuum stresses along the directions parallel to the plates are equal to the energy density. We have checked that the boundary induced VEV in the energy-momentum tensor obeys the covariant continuity equation and its trace is expressed in terms of the photon condensate as $\langle T_{\mu}^{\mu}\rangle_{\rm b} = -\frac{D-3}{16\pi}\langle F_{\mu\sigma}F^{\mu\sigma}\rangle_{\rm b}$, where the index $b$ means "boundary- induced". For $D \geq 4$ the boundary induced contribution in the vacuum energy density is negative for the perfect conductor boundary condition and positive for confining boundary condition. In $D = 3$ spatial dimensions the electromagnetic field is conformally invariant and the vacuum energy-momentum tensor in the interbrane region is the same for both boundary conditions. For $D = 3$ the boundary-induced VEV in the energy-momentum tensor vanishes in the region between the rigth brane and AdS horizon. In the region between the left brane and the AdS boundary the expressions for the vacuum energy-momentum tensor for two types of boundary conditions are different and that is a consequence of different interactions with the AdS boundary.

Based on the expressions for the normal stress we have investigated the Casimir forces acting

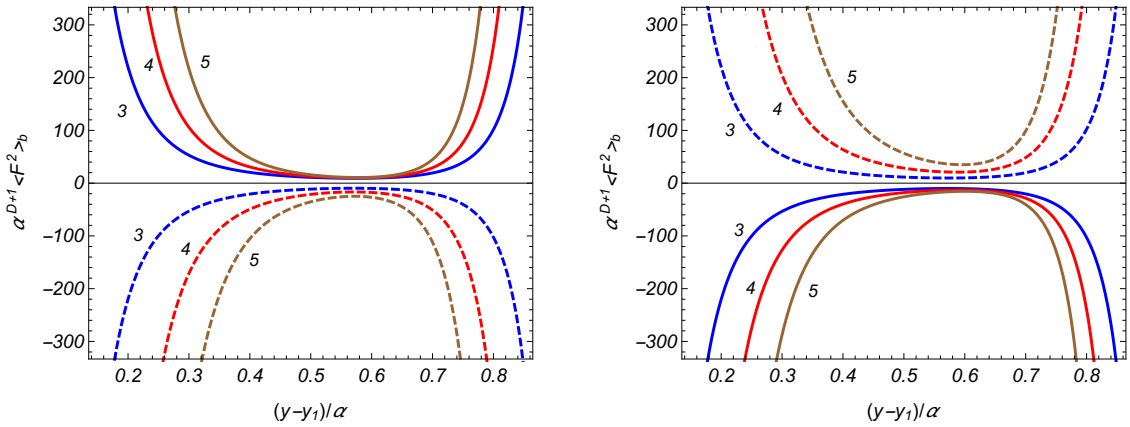


Figure 2: The boundary-induced contributions in the VEVs of the electric (full curves) and magnetic (dashed curves) fields squared in the region between the plates as functions of the distance from the plate at $z = z_1$. The left and right panels correspond to the perfect conductor and confining boundary conditions respectively. The numbers near the curves are the values of the spatial dimension $D$.

on the plates. They are decomposed into the self-action and interaction contributions. The first parts come from the single plate contributions in the normal stress and for $D \neq 3$ require an additional renormalization because of the surface divergences. The $z$-projections of the interaction forces are presented as

$$f_j^{(\mathrm{int})} = -\frac{(D-1)a^{-1-D}z_j^{D+1}}{2^D\pi^{D/2}\Gamma(D/2)}\partial_{z_j}\int_0^\infty du\, u^{D-1}\ln\left|1-\frac{I_\nu(uz_1)K_\nu(uz_2)}{I_\nu(uz_2)K_\nu(uz_1)}\right|, \tag{7}$$

where $\Gamma(s)$ is the Euler gamma function, the indices $j = 1$ and $j = 2$ correspond to the forces acting on the left and right plates. The order of the modified Bessel functions is defined as $\nu = D/2 - 1$ and $\nu = D/2 - 2$ for the perfect conductor and confining boundary conditions, respectively. For $D = 3$ the self-action forces are finite. The force on the right plate is directed toward the AdS boundary for both boundary conditions. The force on the left plate for the confining boundary condition is directed toward the AdS horizon. For the perfect conductor boundary condition the force on the left plate is directed towards the AdS horizon for small separations and towards the AdS boundary for large separations. For general $D$ and at small separations between the plates, compared to the AdS curvature radius, the leading terms in the Casimir forces coincide with that for the plates in the Minkowski bulk. This is a consequence of the fact that at small separations the contribution of the vacuum fluctuations with small wavelengths dominates in the vacuum forces and the influence of gravity on those fluctuations is weak. At separations larger than the AdS curvature scale the gravity essentially changes the behavior of the Casimir forces: considered as functions of the interplate separation, the forces decay exponentially in contrast to the power-law decay for the Minkowski bulk. Another feature differing the AdS and Minkowski bulks is that the forces on the left and right plates differ.

The results obtained can be directly used for the investigation of quantum vacuum effects in $Z_2$-symmetric braneworlds of the Randall-Sundrum type. The boundary conditions on the branes are dictated by the $Z_2$ symmetry and are reduced to the perfect conductor boundary condition for odd fields with respect to the $Z_2$-reflection and to the confining boundary condition for even fields. The expressions for both the local and global characteristics of the vacuum state

are obtained from those we have discussed. One can consider also the situation when the $Z_2$-parities of the fields on the branes are different. In this case we have the perfect conductor boundary condition on one brane and the confining boundary condition on the other. The corresponding mode functions have been derived and the evaluation procedure for the VEVs is similar to that we have described in this chapter.

In **Chapter 4** we investigate the combined effects of boundaries, topology and of the magnetic flux on the ground state mean charge and current densities for a fermionic field in two-dimensional conical rings with arbitrary values of the angle deficit. For planar rings, the corresponding VEVs have been discussed in [7]. For the background geometry under consideration the (2+1)-dimensional line element is given by

$$ds^2 = g_{\mu\nu}dx^\mu dx^\nu = dt^2 - dr^2 - r^2 d\phi^2 \, , \tag{8}$$

where the cylindrical spatial coordinates $r$ and $\phi$ vary in the ranges $r \geqslant 0$ and $0 \leqslant \phi \leqslant \phi_0$. The special case $\phi_0 = 2\pi$ corresponds to the (2+1)-dimensional Minkwoski spacetime described in cylindrical coordinates. For $\phi_0 < 2\pi$, the line element describes a cone with planar angle deficit $2\pi - \phi_0$ and with the apex at $r = 0$. It serves as a simplified model for $D = 3$ cosmic strings with planar angle deficit determined by the linear mass density. We assume that the fermionic field is confined in the region $a \leq r \leq b$ and there is constant background gauge field $A_\mu = (0, 0, A)$. The presence of the gauge field modifies the general phase in the quasi-periodicity condition from $\chi$ to $\alpha = \chi + eA/q$. On the edges of the ring the boundary conditions $(1 + i\lambda_r n_\mu \gamma^\mu)\, \psi(x) = 0$, $r = a, b$, are imposed, where $n_\mu$ is the inward pointing unit vector normal to the boundary, $\gamma^\mu$ is the corresponding Dirac matrix and the parameters $\lambda_a$ and $\lambda_b$ take the values $\pm 1$. In the special case $(\lambda_a, \lambda_b) = (1, 1)$ they are reduced to the standard MIT bag boundary condition (infinite mass boundary condition in the context of 2D fermionic systems). In front of the mass term in the Dirac equation we have added an additional parameter $s = \pm 1$ that corresponds to two inequivalent irreducible representations of the Clifford algebra in (2+1)-dimensional spacetime. The allowed values of the radial quantum number $\gamma$ in the fermionic modes depend on the specific boundary condition and are roots of an equation containing combinations of cylindrical functions. For whole family of boundary conditions, we have considered, the vacuum state is stable and for all the roots $\gamma^2 \geq -m^2$, where $m$ is the mass of the fermionic field. For fields with $(s, \lambda_a, \lambda_b) = (\pm 1, \pm 1, \pm 1)$ all the eigenvalues for $\gamma$ are real. In the remaining cases, depending on $b/a$ and $ma$, purely imaginary eigenvalues $\gamma = i\eta/a$, $0 < \eta < ma$, may appear corresponding to bound states. For half-integer values of the parameter $\alpha$ and under the condition $\lambda_a = -\lambda_b$ there is also a zero mode with the value of the total angular momentum equal to $-\alpha$.

The VEVs of the charge and current densities are evaluated by using the corresponding mode sums over the bilinear products of the mode functions. The VEV for the radial current vanishes and the contribution of the modes with positive $\gamma$ to the charge and azimuthal current densities is derived. In the presence of the bound state or the zero mode, the corresponding contributions should be added to the corresponding expression. The charge and azimuthal current densities on the ring edges are connected by simple relation $\langle j_\phi \rangle_{r=u} = \lambda_u n_u \langle j^0 \rangle_{r=u}$, $u = a, b$, that is valid for whole family of boundary conditions. For half-integer values of $\alpha$ the charge and current densities vanish for the boundary conditions with $\lambda_a = \lambda_b$. For the conditions with $\lambda_a = -\lambda_b$ the only nonzero contribution comes from the zero mode. In the latter case the charge and current densities are discontinuous functions of $\alpha$ (in particular, of the magnetic flux enclosed by the ring) at half-integer values of that parameter.

In the expression for the VEVs there is a summation over the eigenvalues of the radial quantum number $\gamma$ which are not given explicitly. The explicit knowledge of those roots is not

required since we apply a variant of the generalized Abel-Plana formula to the corresponding series to obtain an integral representation. As a result, the VEVs are presented as a sum of the term corresponding to the VEV in the conical geometry with a single boundary at $r = a$ and the contribution induced by the second edge at $r = b$. The former part is further decomposed into the boundary-free and edge induced contributions. As a general rule, the modulus of both the charge and current densities increase with increasing planar angle deficit (with increasing $q$). Depending on the boundary condition, determined by the set $(\lambda_a, \lambda_b)$, the charge and current densities are mainly located near the inner or outer edge. We have demonstrated that the behavior of the VEVs as functions of the mass can be essentially different for fields with $s = +1$ and $s = -1$. In the former case and for the boundary condition with $(\lambda_a, \lambda_b) = (1, 1)$ the absolute values of the charge and current densities decrease with increase of the field mass. In the case $s = -1$ and for the same boundary condition, the absolute values for both the charge and current densities increase with initial increase of the mass. After taking the maximum value, as expected, they tend to zero for large masses.

In two spatial dimensions the fermionic mass term breaks both the parity and time reversal invariances. $P$- and $T$-symmetric fermionic models are constructed considering the set of two fields, $\psi_{(+1)}$ and $\psi_{(-1)}$, with the same masses realizing two inequivalent irreducible representations of the Clifford algebra. If in addition to the masses, the phases in the periodicity condition along the azimuthal direction and the boundary conditions on the edges for the fields $\psi_{(+1)}$ and $\psi_{(-1)}$ are the same then the total charge density vanishes, whereas the total current density doubles. In the effective low-energy theory for electronic subsystem of graphene, the fields $\psi_{(+1)}$ and $\psi_{(-1)}$ correspond to two inequivalent points of the Brillouin zone (valley degrees of freedom) and the obtained results can be applied for the investigation of the charge and current densities induced by Aharonov-Bohm magnetic flux in graphitic cones. Two inequivalent values of the phase $2\pi\chi$ realized in graphitic cones correspond to $\pm 2\pi/3$. It is of interest to note that the valley-dependent gap generation mechanisms create different masses for the fields $\psi_{(+1)}$ and $\psi_{(-1)}$ and, as a result, nonzero net charge density appears. This breaks the time-reversal symmetry.

## CONCLUSIONS

1. For a charged scalar field with general curvature coupling parameter and obeying Robin boundary conditions on two branes in locally AdS spacetime with partly compactified spatial dimensions the quantum fluctuations induce a surface energy-momentum tensor on the branes. For an observer living on the brane it corresponds to a gravitational source of the cosmological constant type. The value and the sign of the induced cosmological constant are determined by the lengths of compact dimensions, by the phases in the periodicity conditions and by the interbrane separation. In the generalized Randall-Sundrum barneworld model the cosmological constant on the visible brane is exponentially suppressed for separations larger than the AdS curvature radius.

2. The VEV of the current density is investigated for a charged fermionic field in the geometry of two branes in background of locally AdS spacetime with toroidally compact dimensions. Nonzero vacuum currents appear along the compact dimensions only. They are decomposed into boundary-free and brane-induced contributions. Both these parts are periodic functions of the magnetic flux enclosed by compact dimensions with the period equal to the flux quantum. Depending on the boundary conditions, the presence

of the branes can either increase or decrease the vacuum current density. For a part of boundary conditions, a memory effect is present in the limit when one of the branes tends to the AdS boundary.

3. Combining the results for two fields corresponding to inequivalent representations of the Clifford algebra, the features of the fermionic current are studied in odd-dimensional parity and time-reversal symmetric models. The corresponding results for three-dimensional spacetime are applied to finite length curved graphene tubes threaded by a magnetic flux. A nonzero current density can also appear in the absence of the magnetic flux if the fields corresponding to two different points of the Brillouin zone obey different boundary conditions on the tube edges.

4. The correlators for the vector potential and for the field strength tensor of the electromagnetic field are evaluated in the geometry of two parallel planar plates in AdS spacetime. Two types of boundary conditions are considered on the plates. The first one is a generalization of perfect conductor boundary condition and the second one corresponds to the confining boundary conditions for gluons in quantum chromodynamics. The VEVs of the photon condensate, of the electric and magnetic fields squared, and of the energy-momentum tensor are investigated.

5. The Casimir forces acting on the plates are decomposed into the self-action and interaction parts. The interaction forces are attractive for both types of boundary conditions. At separations between the plates larger than the curvature radius of the background geometry they decay exponentially as functions of the proper distance. The self-action force per unit surface of a single plate does not depend on its location and depending on the boundary condition and on the number of spatial dimensions can be either attractive or repulsive with respect to the AdS boundary. Applications are given in $Z_2$-symmetric braneworld models of the Randall-Sundrum type for vector fields with even and odd parities.

6. The combined effects of boundaries and topology on the vacuum charge and current densities are investigated for a massive 2D fermionic field confined on a conical ring threaded by a magnetic flux. Different types of boundary conditions on the ring edges are considered for fields realizing two inequivalent irreducible representations of the Clifford algebra. The edge contributions to the VEVs are explicitly extracted and their behavior in various asymptotic limits is considered. On the ring edges the azimuthal current density is equal to the charge density or has an opposite sign. The absolute values of the charge and current densities increase with increasing planar angle deficit. Depending on the boundary conditions, the VEVs are continuous or discontinuous at half-integer values of the ratio of the effective magnetic flux to the flux quantum. The discontinuity is related to the presence of the zero energy mode. The charge and current densities are studied in parity and time-reversal symmetric fermionic models. If the boundary conditions and the phases in quasiperiodicity conditions for separate fields realizing the irreducible representations of the Clifford algebra are the same the total charge density vanishes. Applications are given to graphitic cones with edges (conical ribbons).

## REFERENCES


1. C. G. Callan, Jr., F. Wilczek, *Infrared behavior at negative curvature.* Nucl. Phys. B **340**, 366 (1990).

2. M. Ammon, J. Erdmenger, *Gauge/Gravity Duality: Foundations and Applications* (Cambridge University Press, Cambridge, England, 2015).

3. J. Zaanen, Y.-W. Sun, Y. Liu, K. Schalm, *Holographic Duality in Condensed Matter Physics* (Cambridge University Press, Cambridge, 2015).

4. R. Maartens, K. Koyama, *Brane-World Gravity.* Living Rev. Relativity **13**, 5 (2010).

5. S. Bellucci, A. A. Saharian, V. Vardanyan, *Fermionic currents in AdS spacetime with compact dimensions.* Phys. Rev. D **96**, 065025 (2017).

6. S. Bellucci, A. A. Saharian, D. H. Simonyan, V. Vardanyan, *Fermionic currents in topologically nontrivial braneworlds.* Phys. Rev. D **98**, 085020 (2018).

7. S. Bellucci, A. A. Saharian, A. Kh. Grigoryan, *Induced fermionic charge and current densities in two-dimensional rings.* Phys. Rev. D **94**, 105007 (2016).


## PUBLICATIONS IN THE TOPIC OF THESIS


1. A. A. Saharian, H. G. Sargsyan, *Induced cosmological constant in brane models with a compact dimension.* Astrophysics **61**, 375-390 (2018).

2. H. G. Sargsyan, *Induced cosmological constant in 2-brane models with a compact dimension.* Arm. J. Phys. **12** (4), 329-343 (2019).

3. A. A. Saharian, A. S. Kotanjyan, A. A. Saharyan, H. G. Sargsyan, *Electromagnetic Casimir densities for planar boundaries in AdS spacetime.* Int. J. Mod. Phys. A **35**, 2040029 (2020), 4 pages.

4. S. Bellucci, A. A. Saharian, H. G. Sargsyan, V. V. Vardanyan, *Fermionic vacuum currents in topologically nontrivial braneworlds: Two-brane geometry.* Phys. Rev. D **101**, 045020 (2020), 23 pages.

5. S. Bellucci, I. Brevik, A. A. Saharian, H. G. Sargsyan, *The Casimir effect for fermionic currents in conical rings with applications to graphene ribbons.* Eur. Phys. J. C **80**, 281 (2020), 22 pages.

## ԱՄՓՈՓԱԳԻՐ

1. Կոմպակտ տարածական չափողականություններ ունեցող անտի դե Սիտերի (AdS) տարածությունում գտնվող երկու բրանների վրա Ռոբինի եզրային պայմանների բավարարող և կորության հետ ընդհանուր գործակցով կապված լիցքավորված սկալյար դաշտի քվանտային ֆլուկտուացիաները բրանների վրա մակածում են մակերևութային էներգիա-իմպուլսի թենզոր: Բրանի վրա ապրող դիտորդի համար այն հանդիսանում է կոսմոլոգիական հաստատունի տիպի գրավիտացիոն աղբյուր: Մակածված կոսմոլոգիական հաստատունի արժեքն ու նշանը որոշվում են կոմպակտ չափողականությունների երկայնքով պարբերականության պայմանների փուլերով և բրանների միջև հեռավորությամբ: Ընդհանրացված Ռանդալ-Սունդրումի բրան-աշխարհների մոդելներում, կոսմոլոգիական հաստատունը տեսանելի բրանի վրա էքսպոնենցիալ փոքր է տարածա-ժամանակի կորության շառավղից մեծ միջբրանային հեռավորությունների համար:

2. Տորաձև կոմպակտ չափողականություններ ունեցող և երկու բրաններ պարունակող AdS տարածությունում ուսումնասիրվել է լիցքավորված ֆերմիոնային դաշտի հոսանքի խտության վակուումային միջինը: Ոչ-զրոյական վակուումային հոսանքներ առկա են միայն կոմպակտ չափողականությունների երկայնքով: Դրանք տարանջատվել են առանց բրանների և բրաններով մակածված բաղադրիչների: Այդ երկու ներդրումներն էլ պարբերական ֆունկցիաներ են կոմպակտ չափողականություններով պարփակված մագնիսական հոսքից՝ մագնիսական հոսքի քվանտին հավասար պարբերությամբ: Եզրային պայմաններից կախված՝ բրանների առկայությունը կարող է ինչպես նվազեցնել, այնպես էլ մեծացնել վակուումային հոսանքի խտությունը: Որոշ եզրային պայմանների դեպքում առկա է «հիշողության» էֆեկտ, երբ բրաններից մեկը ձգտում է AdS սահմանին:

3. Համադրելով Քլիֆորդի հանրահաշվի երկու ոչ համարժեք ներկայացումների համար ստացված արդյունքները, ուսումնասիրվել են ֆերմիոնային հոսանքի առանձնահատկությունները տարածական և ժամանակային արտացոլումների նկատմամբ համաչափ տարածական զույգ չափողականությամբ մոդելներում: Երկչափ տարածության համար ստացված արդյունքները կիրառվել են վերջավոր երկարությամբ, մագնիսական հոսք պարփակող կորացած գրաֆենային խողովակների համար: Ոչ զրոյական հոսանքի խտություն կարող է ի հայտ գալ նաև մագնիսական հոսքի բացակայության դեպքում, եթե Բրիլյուենի զոնայի երկու կետերին համապատասխանող դաշտերը խողովակի եզրերին բավարարում են տարբեր եզրային պայմանների:

4. Երկու զուգահեռ թիթեղներ պարունակող AdS տարածությունում հետազոտվել են էլեկտրամագնիսական դաշտի վեկտորական պոտենցիալի և դաշտի թենզորի վակուումային կորելյատորները: Թիթեղների վրա դիտարկվել են երկու տիպի եզրային պայմաններ: Դրանցից առաջինը իդեալական հաղորդչի եզրային պայմանի ընդհանրացումն է, իսկ երկրորդը համապատասխանում է քվանտային քրոմոդինամիկայում գլյուոնները սահմանափակող եզրային պայմանին: Ուսումնասիրվել են ֆոտոնային կոնդենսատի, էլեկտրական և մագնիսական դաշտերի քառակուսիների և էներգիա-իմպուլսի թենզորի վակուումային միջինները:

5. Թիթեղների վրա ազդող Կազիմիրի ուժերը տարանջատվել են ինքնազդեցության և փոխազդեցության բաղադրիչների: Երկու տիպի եզրային պայմանների դեպքում էլ

փոխազդեցության ուժերը ձգողական են: Դրանք էքսպոնենցիալ նվազում են, երբ թիթեղների միջև հեռավորությունը մեծ է ֆոնային տարածության կորության շառավղից: Թիթեղի միավոր մակերեսի վրա ազդող ինքնազդեցության ուժը կախված չէ թիթեղի դիրքից և, կախված եզրային պայմանից ու կոմպակտ չափողականությունների քանակից, կարող է AdS սահմանի նկատմամբ լինել ինչպես ձգողական, այնպես էլ վանողական: Արդյունքները կիրառվել են Ռանդալ-Սունդրումի տիպի $Z_2$-համաչափ բրան աշխարհների մոդելներում՝ զույգ և կենտ զույգությամբ վեկտորական դաշտերի համար:

6. Ուսումնասիրվել է եզրերի և տոպոլոգիայի համատեղ ազդեցությունը կոնական օղակի վրա տեղայնացված զանգվածեղ ֆերմիոնային դաշտի լիցքի և հոսանքի վակուումային խտությունների վրա՝ օղակով թափանցող մագնիսական հոսքի առկայությամբ: Քլիֆորդի հանրահաշվի երկու ոչ համարժեք չբերվող ներկայացումներն իրականացնող դաշտերի համար դիտարկվել են տարբեր սահմանային պայմաններ օղակի եզրերում: Վակուումային միջիններում բացահայտ կերպով առանձնացվել են եզրերի ներդրումները և ուսումնասիրվել է դրանց վարքը տարբեր ասիմպտոտական սահմաններում: Օղակի եզրերի վրա ազիմուտային հոսանքի խտությունը հավասար է լիցքի խտությանը կամ ունի հակառակ նշան: Լիցքի և հոսանքի խտությունների բացարձակ արժեքները աճում են հարթ անկյան դեֆիցիտի աճին զուգընթաց: Էֆեկտիվ մագնիսական հոսքի և հոսքի քվանտի հարաբերության կիսամբողջ արժեքների դեպքում վակուումային միջինները ընդհատ կամ անընդհատ են՝ կախված եզրային պայմաններից: Ընդհատությունը պայմանավորված է զրոյական էներգիա ունեցող մոդայով: Լիցքի և հոսանքի խտություններն ուսումնասիրվել են տարածական և ժամանակային արտացոլումների նկատմամբ համաչափ ֆերմիոնային մոդելներում: Եթե եզրային պայմաններն ու քվազիպարբերականության պայմանների փուլերը Քլիֆորդի հանրահաշվի տարբեր չբերվող ներկայացումներն իրականացնող դաշտերի համար նույնն են, ապա գումարային լիցքի խտությունը հավասարվում է զրոյի: Բերված են կիրառություններ գրաֆենային կոների (կոնական ժապավենների) համար:

**САРГСЯН АЙК**
**НЕКОТОРЫЕ ЭФФЕКТЫ КВАНТОВОЙ ТЕОРИИ ПОЛЯ ВО ВНЕШНЕМ ГРАВИТАЦИОННОМ ПОЛЕ**

1. Квантовые флуктуации заряженного скалярного поля с общим параметром связи с кривизной и с граничными условиями Робина индуцируют поверхностный тензор энергии-импульса на бранах в локально AdS пространстве-времени с частично компактифицированными пространственными измерениями. Для наблюдателя, живущего на бране это соответствует гравитационному источнику типа космологической постоянной. Величина и знак индуцированной космологической постоянной определяются длинами компактных измерений, фазами в условиях периодичности и межбранным расстоянием. В обобщенных моделях мира на бране Рандалла-Сундрума космологическая постоянная на видимой бране экспоненциально мала для расстояний, превышающих радиус кривизны пространства-времени.

2. Исследовано вакуумное среднее плотности тока заряженного фермионного поля в геометрии с двумя бранами на фоне локально AdS пространства-времени с тороидально компактными измерениями. Ненулевые вакуумные токи возникают только вдоль компактных измерений. В соответствующих выражениях явно выделены части, индуцированные бранами. Они являются периодическими функциями магнитного потока, заключенного компактными измерениями, с периодом, равным кванту потока. В зависимости от граничных условий наличие бран может как увеличивать, так и уменьшать плотность вакуумного тока. Для некоторых граничных условий присутствует эффект памяти в пределе, когда одна из бран стремится к границе AdS.

3. Объединив результаты для двух полей, соответствующих неэквивалентным представлениям алгебры Клиффорда, исследованы характеристики фермионного тока в нечетномерных P- и T-симметричных моделях. Соответствующие результаты для трехмерного пространства-времени применяются к искривленным графеновым трубкам конечной длины, проницаемым магнитним потоком. Ненулевая плотность тока может появиться и в отсутствии магнитного потока, если поля, соответствующие двум разным точкам зоны Бриллюэна, подчиняются различным граничным условиям на краях трубки.

4. Исследованы корреляторы для векторного потенциала и тензора электромагнитного поля в геометрии двух параллельных плоских пластин в пространстве-времени AdS. На пластинах рассматриваются два типа граничных условий. Первый является обобщением граничного условия идеального проводника, а второй соответствует граничному условию конфайнмента для глюонов в квантовой хромодинамике. Исследованы значения вакуумных средних фотонного конденсата, квадрата электрического и магнитного полей, а также тензора энергии-импульса.

5. Действующие на пластины силы Казимира разлагаются на вклады самодействия и взаимодействия. Силы взаимодействия являются притягивающими для обоих типов граничных условий. На расстояниях между пластинами, превышающими радиус кривизны фоновой геометрии, эти силы уменьшается со растущим собственным

расстоянием по экспоненциальному закону. Сила самодействия на единицу поверхности пластины не зависит от ее положения и в зависимости от граничного условия и размерности пространства может быть как притягивающей, так и отталкивающей по отношению к границе AdS. Приведены приложения к $Z_2$ - симметричным моделям мир на бране Рандалла-Сундрума для четных и нечетных векторных полей отнсоительно $Z_2$-отражения.

6. Исследовано совместное влияние границ и топологии на плотности вакуумного заряда и тока для массивного фермионного поля, локализованного на коническом кольце, через которое проникает магнитный поток. Рассмотрены различные типы граничных условий на краях кольца для полей, реализующих неэквивалентные неприводимые представления алгебры Клиффорда. В вакуумных средних явным образом выделены вклады границ и исследовано их поведение в различных асимптотических областях параметров. На краях кольца азимутальная плотность тока либо равна плотности заряда, либо противоположна ей. Абсолютные значения плотности заряда и тока увеличиваются с увеличением дефицита планарного угла. В зависимости от граничных условий вакуумные средние являются либо непрерывными, либо прерывными при полуцелых значениях отношения эффективного магнитного потока к кванту потока. Прерывность связана с наличием моды с нулевой энергией. Изучены плотности заряда и тока в P- и T-симметричных фермионных моделях. Если граничные условия и фазы в условиях квазипериодичности для отдельных полей, реализующих неприводимые представления алгебры Клиффорда совпадают, то полная плотность заряда обращается в нуль. Описаны применения для графеновых конусов с краями (конические ленты).