Review of Vazgen Mikayelyan's dissertation "On the convergence of series with respect to Franklin system its generalization" submitted for the degree of Candidate of Physical and Mathematical Sciences A. 01.01 -Mathematical Analysis

The dissertation is devoted to current topics in the Fourier series in relation to the Franklin system. Cheselski began an in-depth study of Franklin systems. Cheselski 's work laid the groundwork for further research in this area. It is known that the Franklin system is base in the class of continuous functions and in the space L_p. Bochkarov proved that this system is an unconditional base in space L_p. Moreover, this system is an unconditional basis in reflexive Orlicz spaces. Next, the development of this system was facilitated by Gevorkyan, where he has proved his fundamental theorems. Generalized Cheselski systems were studied by Gevorkyan, Kamont, and their students.

The first chapter of the dissertation is devoted to the divergence to infinity of series with respect to Franklin system on the set of positive measure . This question is based on Luzin's well-known problem, which was raised in 1915 about the divergence of trigonometric series to infinity. It is well known that Riemann means of a trigonometric series cannot be a divergence to the infinity in a set of positive measures. Privalov constructed the example of a trigonometric series, which by the Abel method is almost everywhere converges to infinity. Menshev has proved that for any given function there exists a trigonometric series that is summable in measure to the given function. For any measurable function, Talalyan found a trigonometric series that is almost everywhere convergent to a given function at the points where the function f is finite. Finally, Koniagin solved the problem Luzin's, namely the following Theorem has been proved by him:

Theorem. Let $S_n(x), n = 0,1,\cdots, x \in [-\pi,\pi]$, be partial sums of a trigonometric series. Then

$$\operatorname{mes}\left\{x \in [-\pi,\pi]: -\infty < \liminf_{n\to\infty} S_n(x) \le \limsup_{n\to\infty} S_n(x) = +\infty\right\} = 0$$

In particular, a trigonometric series cannot be convergence to infinity on a positive measure set.

A similar problem with the Haar and Walsh systems was solved by Talalian and Arutunian. However, there is uniformly bounded orthonormal system and a rearrangement for which series converges to infinity on the set of positive measure. Pogosian established the existence of an analogous theorem for any complete orthonormal system. Finally, the analogy of Koniagin theorem for Franklin systems was proved by Gevorkyan.

The first chapter proves a similar Gevorkyan's theorem for generalized Franklin systems. In particular, the following is proved:

Theorem 1.1.6. For the generalized Franklin systems, the following equality is true:

$$\mu\left(\left\{x \in [0,1]: -\infty < \liminf_{n} \sigma_n(x) \leqslant \limsup_{n} \sigma_n(x) = +\infty\right\}\right) = 0.$$

Gevorkyan proves that divergence to infinity does not occur along the 2^k sequence. In particular, the following is proved.

Theorem. Let S be partial sum of a series with respect to Franklin system. Then

$$\mu\left(\left\{x \in [0,1]: \lim_{k\to\infty} S_{2^k}(x) = +\infty\right\}\right) = 0.$$

In the first chapter of the dissertation, the issue is solved: to characterize all the classes of subsequences that will validate the Gevorkyan Theorem. In the dissertation successfully solved the mentioned issue and proved the necessary and sufficient conditions so that the partial sums of the Fourier series with respect to the Franklin system were not divergence to the infinity on a set of positive measure.

Theorem 1. 1. 2 Let $\sup_{k\in\mathbb{N}} \frac{n_{k+1}}{n_k} < +\infty$. Then $\mu\left(\left\{x \in [0,1]: \lim_{k\to\infty} \sigma_k(x) = +\infty\right\}\right) = 0.$

Theorem 1.1.3 Let $\sup_{k\in\mathbb{N}} \frac{n_{k+1}}{n_k} = +\infty$. Then there exist series with respect to Franklin system, such that $\lim_{k\to\infty} \sigma_k(x) = +\infty$ a. e. on [0,1].

A basis $\{x_i\}_{i=1}^{\infty}$ for a Banach space X is said to be boundedly complete if whenever $\{a_i\}_{i=1}^{\infty}$ is a sequence of scalars for which $\sup_n \|\sum_{i=1}^{n} a_i x_i\| < +\infty$, then $\sum_{i=1}^{\infty} a_i x_i$ converges.

The semi-normalized basis $\{x_i\}_{i=1}^{\infty}$ for the Banach space X is said to be monotonically boundedly complete if whenever $\{a_i\}_{i=1}^{\infty}$ is a sequence of scalars which decreases monotonically to zero and for which $\sup_n \|\sum_{i=1}^{n} a_i x_i\| < +\infty$, then $\sum_{i=1}^{\infty} a_i x_i$ converges.

The following theorem has been proved by Holub

Theorem The Schauder system is a monotonically boundedly complete basis for $C[0,1]$.

He set the next task: Would the Haar and Franklin systems be the monotonically boundedly complete basis? Kadec has proved that the Haar system is a monotonically boundedly complete basis for $L_1[0,1]$. In the first chapter of the dissertation has been proved that the Franklin system is a monotonically boundedly complete basis for $C[0,1]$ and $L_1[0,1]$.

The Gibbs phenomenon, discovered by Henry Wilbraham (1848) and rediscovered by J. Willard Gibbs (1899) and is the peculiar manner in which the Fourier series of a piecewise continuously differentiable periodic function behaves at a jump discontinuity. The nth partial sum of the Fourier series has large oscillations near the jump, which might increase the maximum of the partial sum above that of the function itself. The overshoot does not die out as n increases, but approaches a finite limit. This sort of behavior was also observed by experimental physicists, but was believed to be due to imperfections in the measuring apparatus. This is one cause of ringing artifacts in signal processing. In the first chapter of the dissertation has been investigated the Gibbs phenomenon for the Franklin system. The Stromberg Wavelet case has also discussed.

It follows from the above that the dissertation is completed at the highest level,

It follows from the above, that the dissertation is devoted to the actual questions of theory of orthogonal series. The dissertation has been made at the highest level. All the results of the dissertation are new and will undoubtedly contribute to the development of theory of orthogonal series.

Therefore, the dissertation **"On the convergence of series with respect to Franklin system its generalization"** satisfies all the requirements and its author **Vazgen Mikayelyan** deserves the degree of Candidate of Physical and Mathematical Sciences A. 01.01 -Mathematical Analysis.

Doctor of Physical and Mathematical Sciences,
Associate Professor at the United Arab Emirates University
Ushangi Goginava

Gog

جامعة الإمارات العربية المتحدة
UNITED ARAB EMIRATES UNIVERSITY
Dept. of Math. & Comp. Sci. - Faculty of Science

June 15, 2021