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Bergische Universität Wuppertal, Chair of Stochastics, Dr. Hayk Asatryan
Gaußstraße 20, 42119 Wuppertal, Germany

Prof. K. Khachatryan
Yerevan State University
Faculty of Mathematics and Mechanics
Chair of Differential Equations
1 Alex Manoogian
Yerevan, 0025
Republic of Armenia

Dr. Hayk Asatryan
School of Mathematics and Natural Sciences
Chair of Stochastics
Gaußstraße 20, 42119 Wuppertal, Germany

Room	FZ.00.20
Phone	+49 (0)202 439 2767
Mail	asatryan@math.uni-wuppertal.de
Internet	https://stochastik.uni-wuppertal.de/index.php?id=5125&L=0
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Review of the Dissertation

"Issues in construction of fixed points for some integral operators with convex nonlinearities"

submitted by Arusyak Sisakyan
for attaining a scientific degree of

Candidate of Physical and Mathematical Sciences

The dissertation is devoted to fixed point construction problems for scalar, vector and discrete nonlinear integral operators arising in several areas of mathematical physics. The peculiarity of the considered operators is that they are locally monotone, non-compact in certain function spaces, critical (i.e., possess trivial fixed points) and their integration domains are unbounded. This makes classical fixed point theorems inapplicable to these operators, hence each of them needs to be investigated separately. Important investigations on nonlinear integral equations in unbounded domains, as highlighted in the introductory section of the dissertation, can be found in the works of V. Vladimirov, I. Volovich, N. Yengibaryan, O. Diekmann, L. Arabadzyan, A. Sergeev, A. Khachatryan and K. Khachatryan. The represented work of A. Sisakyan investigates new classes of equations with analogues properties.

The dissertation consists of an introduction, 3 chapters, a summary and a bibliography (totally 83 pages). In the introductory section A. Sisakyan presents relevant results by other authors, as well as states the main results of her work.

The first part of the first chapter is devoted to the construction of non-trivial solutions and investigation of their asymptotic behavior for a class of integral equations generated by a Urysohn-type non-compact operator. Assuming a Hammerstein-type integral operator with a sum-difference kernel and with a power-law nonlinearity to be a local minorant for an Urysohn-type monotone operator, the author managed under certain technical conditions to construct a nonnegative non-trivial solution to

the equation and even to find its limit at infinity (see Theorem 1.1). The rest of the first chapter is devoted to the investigation of a class of integral equations with a power-law nonlinearity on the whole axis. These equations arise in the kinetic theory of gases. Combining special factorization methods with construction methods for invariant conic segments of corresponding nonlinear operators, the author has constructively confirmed the existence of an one-parameter family of non-trivial bounded signed solutions (see Theorems 1.3 and 1.4).

The second chapter of the dissertation is devoted to a nonlinear system of singular integral equations, arising in the dynamic theory of open and closed p-adic wires. Enhancing some iteration methods, as well as certain methods from the theory of positive matrices and using some limit theorems from real analysis, the author constructed a signed solution to the considered system and investigated its asymptotic behavior (see Theorem 2.2).

The third chapter treats an infinite system of algebraic equations with convex nonlinearities, generated by Toeplitz-Hankel matrices. Systems of such kind can be found in discrete problems of the p-adic mathematical physics. The continuous analogues of those systems are thoroughly investigated by K. Khachatryan. By means of certain estimates a theorem on existence of a nonnegative solution in the space of bounded sequences is proven. Moreover, the asymptotic behavior of the constructed solution at the infinity is investigated (see Theorem 3.1).

At the end of each chapter the author provides with interesting examples of nonlinearities and kernel functions satisfying the conditions of preceding theorems.

In my opinion, the dissertation would profit if the author completed the following additional investigations:

1. The solvability of equation (0.6) for even values of p,
2. Implementation and discussion of numerical methods for the constructed solutions.

Passing to the overall evaluation of the dissertation, I should mention that it contains solid mathematical results which are important contributions in the theory of nonlinear integral equations. The obtained results are published in 4 scientific journals, 2 of which are scopus indexed. The author's abstract correctly reflects the dissertation.

I endorse that the dissertation of A. Sisakyan fulfills the requirements of the Qualifying Committee of the Republic of Armenia, and its author deserves to obtain the scientific degree of Candidate of Physical and Mathematical Sciences in the field of Differential Equations and Mathematical Physics (A.01.02).

Sincerely,

Hayk Asatryan
School of Mathematics and Natural Sciences
University of Wuppertal
Wuppertal, Germany