Ա.Ի. ԱԼԻԽԱՆՅԱՆԻ ԱՆՎԱՆ ԱԶԳԱՅԻՆ ԳԻՏԱԿԱՆ ԼԱԲՈՐԱՏՈՐԻԱ
(ԵՐԵՎԱՆԻ ՖԻԶԻԿԱՅԻ ԻՆՍՏԻՏՈՒՏ)

Ավետիսյան Մանե Երվանդի

ՎԱԺԵԼԻ ՈՒՆԻՎԵՐՍԱԼՈՒԹՅՈՒՆԸ ԵՎ ՆՐԱ ԿԻՐԱՌՈՒԹՅՈՒՆՆԵՐԸ

Ա.04.02 - «Տեսական Ֆիզիկա» մասնագիտությամբ
ֆիզիկամաթեմատիկական գիտությունների թեկնածուի
գիտական աստիճանի հայցման ատենախոսության

ՍԵՂՄԱԳԻՐ

ԵՐԵՎԱՆ – 2022

A. I. ALIKHANYAN NATIONAL SCIENCE LABORATORY

(YEREVAN PHYSICS INSTITUTE)

Mane Avetisyan

VOGEL'S UNIVERSALITY AND ITS APPLICATIONS

SYNOPSIS

of Dissertation in 01.04.02-Theoretical Physics presented for the degree of a candidate in physical and mathematical sciences

YEREVAN – 2022

Ատենախոսության թեման հաստատվել է Ա. Ի. Ալիխանյանի անվան Ազգային Գիտական Լաբորատորիայի (ԵրՖի) գիտական խորհրդում:

Գիտական ղեկավար՝
Ֆիզմաթ. գիտ. դոկտոր Ռուբեն Լևոնի Մկրտչյան (ԱԱԳԼ)

Պաշտոնական ընդդիմախոսներ՝
Ֆիզմաթ. գիտ. դոկտոր Ռուբիկ Հրաչիկի Պողոսյան (ԱԱԳԼ)
Ֆիզմաթ. գիտ. դոկտոր Անդրեյ Դմիտրիի Միրոնով (ԼՖԻ)

Առաջատար կազմակերպություն՝
Միջուկային հետազոտությունների միացյալ ինստիտուտ (ՄՀՄԻ), Դուբնա, Ռուսաստանի Դաշնություն

Ատենախոսության պաշտպանությունը կայանալու է 2022 թ. հուլիսի 6-ին ժամը 14:00-ին, ԱԱԳԼ-ում գործող ԲՈԿ-ի 024 «ֆիզիկայի» մասնագիտական խորհրդում (Երևան, 0036, Ալիխանյան եղբ. փ. 2):
Ատենախոսությանը կարելի է ծանոթանալ ԱԱԳԼ-ի գրադարանում:
Սեղմագիրը առաքված է 2022 թ. մայիսի 23-ին:

Մասնագիտական խորհրդի գիտական քարտուղար՝
Ֆիզմաթ. գիտ. դոկտոր Հրաչյա Հովհաննեսի Մարուքյան

The subject of the dissertation is approved by the scientific council of the A.I. Alikhanyan National Science Laboratory (YerPhI)

Scientific supervisor:
Doctor of ph-math. sciences Ruben Mkrtchyan (AANL)

Official Opponents:
Doctor of ph-math. sciences Rubik Poghossian (AANL)
Doctor of ph-math. sciences Andrei Mironov (LPI)

Leading Organization:
Joint Institute of Nuclear Research, Dubna, Russia

The defense will take place on the 6th of July, 2022 at 14:00 during the “Physics” professional council’s session of SCC 024 acting within AANL (2 Alikhanyan Brothers str., 0036 - Yerevan)
The dissertation is available at the AANL library.
The Synopsis is sent out on the 23th of May, 2022.

Scientific secretary of the special council:
Doctor of ph-math. sciences Hrachya Marukyan

Abstract

In this thesis we present developments related to the universal description of the simple Lie algebras in two main directions.

The first one is devoted to the representation theory of the simple Lie algebras. Specifically we present recent results, which include new universal quantum dimension formulae in Vogel's universal description, the discovery of a new property of those formulas called linear resolvability, and the derivation of a four-by-four non-uniqueness factor in the scope of the uniqueness problem of universal dimension formulae.

In the second part of the thesis we demonstrate applications of Vogel's description to the study of a physical theory. Namely, we explicitly formulate the refined Chern-Simons theories on S^3 for each of the simple gauge groups, including the exceptional ones. We also present the corresponding partition functions in a universal-like form. That provide us with a new tool for the investigation of possible Chern-Simons/ topological strings dualities for all gauge groups.

Relevance of the scientific research

Vogel's universal approach to the simple Lie algebras, first introduced in [1], is a powerful and attractive tool both for mathematicians and theoretical physicists.

First of all, it provides a uniform parametrization of the simple Lie algebras, allowing to unify those innately discrete objects by providing a way of treating them as different points in a projective plane. The corresponding coordinates, i.e. triplets of values, defining each of the algebra, is given in Table 1, which is called Vogel's table.

To outline the origin of this uniform parametrization we look at the symmetric square of the adjoint representation of a simple Lie algebra [1]:

$$S^2\mathfrak{g} = 1 \oplus Y_2(\alpha) \oplus Y_2(\beta) \oplus Y_2(\gamma)$$

Note that for each of the simple Lie algebra the decomposition above is the same.

Then let $2t$ denote the eigenvalue of the second Casimir operator on the adjoint representation $\mathfrak{g}$ and the eigenvalues of the same operator on representations $Y_2()$ be $4t-2\alpha, 4t-2\beta, 4t-2\gamma$, correspondingly. In this way we define α, β, γ (Vogel's) parameters. It can be proved [1] that with these definitions $\alpha+\beta+\gamma=t$. According to the definitions, the entire theory is invariant with respect to a rescaling of these parameters (which corresponds to the rescaling of the invariant scalar product in algebra), and with respect to the permutation of the universal (or, Vogel's) parameters

Root system	Lie algebra	α	β	γ	$t=h^\vee$
A_n	$\mathfrak{sl}_{n+1}$	-2	2	$(n+1)$	$n+1$
B_n	$\mathfrak{so}_{2n+1}$	-2	4	$2n-3$	$2n-1$
C_n	$\mathfrak{sp}_{2n}$	-2	1	$n+2$	$n+1$
D_n	$\mathfrak{so}_{2n}$	-2	4	$2n-4$	$2n-2$
G_2	$\mathfrak{g}_2$	-2	$10/3$	$8/3$	4
F_4	$\mathfrak{f}_4$	-2	5	6	9
E_6	$\mathfrak{e}_6$	-2	6	8	12
E_7	$\mathfrak{e}_7$	-2	8	12	18
E_8	$\mathfrak{e}_8$	-2	12	20	30

Table 1: Vogel's Table

α, β, γ. In essence, these parameters belong to a projective plane, which is factorized w.r.t. its homogeneous coordinates and is called Vogel's plane, see Figure 1.

As is seen, it demonstrates the points from Vogel's table. Also, it includes some additional points and lines studied by Landsberg, Manivel, Westbury, and Mkrtchyan, namely, the line corresponding to $D_{(2,1,\lambda)}$ superalgebras, the $3d$ line, which passes through the $sl(2)$ point, etc.

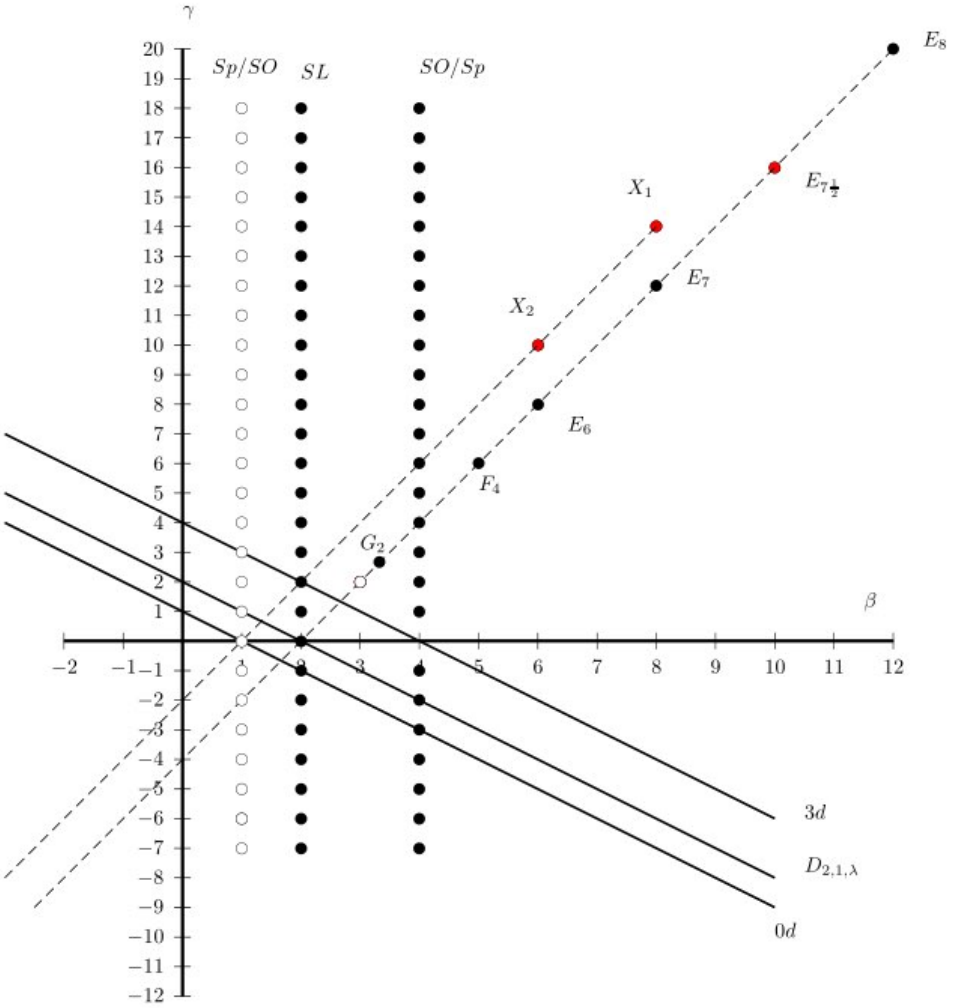

Figure 1: Vogel's Plane

This parameterization of the simple Lie algebras happens to be very convenient and useful. In particular, the existence of the so-called universal formulae for several objects appearing both in the representation theory of the simple Lie algebras and physical theories built upon the symmetries corresponding to the simple groups is made possible due to this parametrization.

As typical examples of universal formulae, those for the dimensions of the adjoint $\mathfrak{g}$ and $Y_2()$ representations are presented below:

$$\dim\mathfrak{g} = -\frac{(2t-\alpha)(2t-\beta)(2t-\gamma)}{\alpha\beta\gamma};$$

$$\dim Y_2(\alpha) = \frac{(2t-3\alpha)\big(\beta-2t\big)\big(\gamma-2t\big)t\big(\beta+t\big)\big(\gamma+t\big)}{\alpha^2\big(\alpha-\beta\big)\beta\big(\alpha-\gamma\big)\gamma};$$

and the other two $Y_2(\beta), Y_2(\gamma)$ representations are obtained by permutations of the parameters [1,2].

Universal formulae have shown their importance even beyond the scope of the representation theory of Lie algebras. E.g. Vogel [1] found a complete decomposition of the third power of the adjoint representation in terms of Universal Lie Algebra, defined by him, and universal dimension formulae for all representations involved. Landsberg and Manivel [2] presented a method that allows the derivation of certain universal dimension formulae for simple Lie algebras and derived those for the Cartan powers of the adjoint, $Y_2(.)$, and their Cartan products. A universal formula for the quantum dimension of the adjoint representation has been found by Westbury [3]. Sergeev, Veselov, and Mkrtchyan have derived [4] a universal formula for generating function for the eigenvalues of higher Casimir operators on the adjoint representation.

Using the universal approach, applications to physics were developed. Particularly the universality of the partition function of Chern-Simons theory on a S^3 sphere [5, 6, 7] was shown. Its connection with q-dimension of the $k\Lambda_0$ representation of affine Kac-Moody algebras [8] was shown, and the universal knot polynomials for 2- and 3-strand torus knots [9, 10, 11, 12] were calculated.

Another application of universal formulae is the derivation of non-perturbative corrections to Gopakumar-Vafa partition function [13, 14] by gauge/string duality from the universal partition function of Chern-Simons theory. This shows the relevance of the

”analytical continuation” of the universal formulae from the points of Vogel’s table to the entire Vogel’s plane.

A completely different direction of development, the Diophantine classification of simple Lie algebras [15] and its connection with the McKay correspondence [16], is also worth mentioning.

The attainments listed above motivate the relevance of developing Vogel's approach and investigating its applications to physical gauge theories.

Aim of the dissertation

One of the aims of this work is the deeper understanding of Vogel's universal description of simple Lie algebras. Another one is the better understanding of the possibility of setting up a dualities between the refined Chern-Simons theories on S^3 and some (refined) topological strings living on specific Calabi-Yau manifolds. More specifically, our purposes were:

- the understanding of the possibility of the extension of the list of universal functions describing characteristics of the simple Lie algebras;
- the investigation of the singularities of universal dimension formulae and their possible classification;
- the understanding of the uniqueness problem of universal formulae;
- the understanding of dualities between the refined Chern-Simons theories on S^3 with arbitrary gauge groups and topological string theories.

Results submitted for defense

1. Derivation of universal dimension and quantum dimension formulae for Cartan products of arbitrary powers of the adjoint g and X_2 representations ($(g^n(X_2)^k$, $k, n \in Z_+$) of the simple Lie algebras. Study of these formulae under permutations of

universal parameters and demonstration that in their stable limits the outputs are quantum dimensions of some representations of the corresponding algebras.

2. Definition of the linear resolvability feature of the universal formulae. Proof that all quantum dimension formulae known so far, including our newly-derived one, are linearly resolvable.

3. Derivation of universal eigenvalues of the second Casimir operator on the Cartan products of arbitrary powers of the adjoint g and X_2 representations.

4. Geometrical interpretation of the universal formulae. Establishment of a correspondence between non-uniqueness factors of universal formulae and geometrical configurations of points and lines. Derivation of a four-by-four non-uniqueness factor using this correspondence.

5. Refinement of the Kac-Peterson identity for the determinant of the symmetrized Cartan matrix. Derivation of an explicit formula for the partition functions of the refined Chern-Simons theory on S^3 with an arbitrary simple gauge group.

6. Universal-like representation of all these partition functions of the refined Chern-Simons theory on S^3 with an arbitrary simple gauge group. This representation aims at a further check of possible Chern-Simons/topological strings dualities for all gauge groups.

The articles, on which the current thesis is based on, are listed in the **Publication list** below in the synopsis presented.

Practical value

First of all, the knowledge on the properties of the symmetries of the simple Lie algebras and their representations is highly used in various fields of study. Some evident ones are: modern biology, chemistry and material science, cryptography, and even music theory. In this thesis we derived a universal formula for a whole series of representations. This formula definitely makes the calculations of the corresponding

dimensions and quantum dimensions considerably easier. We made it available online in the Wolfram Mathematica's notebook archive (https://notebookarchive.org/on-ad-n-x_2-k-series-of-universal-quantum-dimensions--2021-01-byv860k/) , so that anyone interested can make use of it.

Second, the geometrical interpretation of the universal formulae developed in this thesis, and the subsequent establishment of a correspondence between non-uniqueness factor and geometrical configurations, can serve as an algebraic coding of a configuration. This might hopefully be used in the modern visualization problems or in the related fields.

The other major part of the thesis is related to the investigation of dualities between physical theories via the tools from Vogel's universality. The revelation of possible dualities of physical theories are essential for the modern understanding of nature. The universality approach used in this thesis might hopefully be used not only to the study of the Chern-Simons/topological strings dualities, but also in other physical theories.

The novelty of the work

The research presented develops Vogel's universal approach to simple Lie algebras by expanding the list of universal representations which has remained unchanged since 2005. It also presents an explicit expression for the partition function of the refined Chern-Simons on S^3 for all simple gauge groups.

Length and structure of the dissertation

The dissertation contains 6 chapters and the bibliography. The first and the last chapters are the Introduction and the Summary, correspondingly. The other four chapters describe our findings.

Content of the dissertation

In chapter 1 (Introduction) we review the material necessary to present our findings. More exactly, it is introductory notions describing Vogel's universality and its state-of-the-art. In chapters 2-5 we explain the background of the solved problem and deliver the findings. In the Summary (chapter 6) we review our findings and discuss the possible directions of research springing out of them.

Chapter 1

We introduce the idea of universality, first used by Vogel at the end of the twentieth century. Then we give an idea of the uniform parametrization of the simple Lie algebras, presenting the Vogel's table along with the so-called Vogel's plane. To give an idea on the origin of Vogel's table and Vogel's plane, we also outline the procedure of this parametrization. After that we demonstrate two basic examples of universal dimension formulae. Next we present the state-of-the-art of the universal approach to the simple Lie algebras. We end this chapter by a short description of the directions which are developed in the thesis.

Chapter 2

We present the first results submitted for defense, namely the new universal formulae for quantum dimensions and Casimir eigenvalues.
We first present the history of derivation of universal formulae and mention that the possibility of the extension of the list of universal dimensions is still an open question. Then we mention the technique, namely the Landberg's and Manivel's one, used for the derivation of our new quantum dimension formulae. The next sections are based on the papers [5, 6] in the publication list. Namely:

- we give an idea on the origin of the X_2 representation;

- we bring the description of the technique, used for the investigation of the scalar products of the highest weights of the Cartan powers of the X_2 representation for all simple Lie algebras;

- we propose a new universal formula for quantum dimensions of arbitrary Cartan powers of the X_2 representation;

- we propose a generalization of that formula to the quantum dimension of the Cartan products of arbitrary powers of the adjoint and X_2 representations;

- we present the results of the study of these new formulae under permutations of the universal parameters;

- we present the derivation of the eigenvalues of the second Casimir operator on these representations and discuss the conformity checks with known results, based on the paper [3] in the publication list.

This chapter is wrapped up with the Appendix C.II, which presents the proof of the main propositions.

Chapter 3

We begin this chapter by introducing the reader to an interesting feature of the universal dimensions, noticed before. Namely, when considering them at the permuted coordinates of the initial special points in Vogel's plane, they (usually) yield some reasonable outputs, which naturally correspond to some other representations of the algebra, associated with the permuted coordinates. Sometimes the permuted coordinates correspond to singular points of the universal formulae. The main result presented in this chapter says that all known quantum dimension formulae, including the $X(x,k,n,\alpha,\beta,\gamma)$ derived in the previous chapter have a feature which allows obtaining finite and reasonable answers at their singular points, associated with those from Vogel's table.

The next sections of this chapter is based on the material from [1,5] in the publication list, namely:

- we formally define the *linear resolvability (LR)* feature;
- we show that all known universal dimension formulae, including the $X(x,k,n,\alpha,\beta,\gamma)$ universal formula are linearly resolvable at the points from Vogel's table;
- we make a conjecture on the interpretation of the outputs of our formula at those singular points;
- we demonstrate an interesting phenomenon of our function, that is yielding two different values at a single singular point associated with the $so(8)$ algebra.

We conclude this chapter with the discussion of the LR feature beyond the points from Vogel's table.

Chapter 4

In this chapter

- we formulate the problem of uniqueness of universal dimension formulae and introduce the notion of a *non-uniqueness factor;*
- we present a geometrical formulation of the uniqueness problem and show that it brings us to a completely new area of mathematics – the theory of configurations of points and lines
- we derive an explicit expression for a four-by-four non-uniqueness factor, making use of a known $(16_3, 12_4)$ configuration of points and lines.

These are based on my [2] publication.
Some interesting configurations, such as the famous Pappus configuration, and a corresponding three-by-three non-uniqueness factor is also discussed in this chapter. The problem of finding a completely symmetrical non-uniqueness factor which will correspond to a realizable $(144_3, 36_{12})$ configuration and will solve the problem of uniqueness of universal dimensions is addressed as well.

Chapter 5

This chapter is addressed to describe our recent results in the scope of the applications of Vogel's universality to the study of dualities between physical theories. It is based on the material from our [1] publication. The following results are presented:

- we generalize the universal partition function of the Chern-Simons theory on S^3 to the *refined* case, and present its explicit expression for an arbitrary gauge group;
- we show that the previously known Krefl-Schwartz representation of the partition function of the refined Chern-Simons on S^3can be generalized to all simply laced algebras;
- we derive universal-like representations of that partition function for all non-simply laced gauge algebras. This presentation makes it possible to derive each of the refined partition functions in a form, suitable for comparing it with the Gopakumar-Vafa partition functions for topological strings.

Chapter 6

This chapter is the summary of the results presented in the thesis.

- New universal formulae for quantum dimensions of $(X_2)^k(g)^n$ representations and for the second Casimir eigenvalues on them have been discovered.

- A new property of all quantum dimension formulae, that is linear resolvability has been revealed.
- A remarkable connection between the universal formulae for the simple Lie algebras and some geometrical configurations of points and lines has been formulated.
- Finally, a step forward has been taken in the understanding of the dualities between the refined Chern-Simons theories based on arbitrary simple gauge groups and some topological strings. More exactly, construction of the refined partition functions for all simple gauge groups, ready to be transformed into Gopakumar-Vafa type partition functions of topological strings, has been carried out.

We also address some possible directions of research springing out of these results:

- the possible derivation of new universal formulae;
- the final solution to the problem of the uniqueness of universal dimension formulae;
- the complete understanding of the Chern-Simons/topological strings dualities for all simple gauge groups.

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Publication list

1. M.Y.Avetisyan and R.L.Mkrtchyan, On linear resolvability of universal quantum dimensions, Journal of Knot Theory and Its Ramifications, Vol. 31, No. 2 (2022) 2250014, https://doi.org/10.1142/S0218216522500146.

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ԱՎԵՏԻՍՅԱՆ ՄԱՆԵ ԵՐՎԱՆԴԻ

ՎԱԺԵԼԻ ՈՒՆԻՎԵՐՍԱԼՈՒԹՅՈՒՆԸ ԵՎ ՆՐԱ ԿԻՐԱՌՈՒԹՅՈՒՆՆԵՐԸ

Ամփոփագիր

Վաժելի ունիվերսալության գաղափարը Լիի պարզ հանրահաշիվներն ու դրանց հիմքով կառուցված ֆիզիկական տեսություններն ուսումնասիրելու այլընտրանքային արդյունավետ գործիք է: Վերջին քսան տարիների ընթացքում դրա միջոցով ուսումնասիրվել ու զարգացում է ստացել Լիի պարզ հանրահաշիվների ունիվերսալ ներկայացումների տեսությունը: Բացի դրանից, այդ ընթացքում ի հայտ է եկել ֆիզիկական տեսությունների միջև հնարավոր դուալությունների ուսումնասիրության հարցում Վաժելի ունիվերսալության մոտեցման նշանակալի ներդրումը:

Այս թեկնածուական ատենախոսության մեջ ներկայացված վերոնշյալ ուղղությունների զարգացմանը միտված արդյունքներ: Թվարկենք դրանք՝

- Նոր ունիվերսալ բանաձևերի դուրսբերում՝ կցված (ad) և X_2 ներկայացումների կամայական Կարտան-աստիճանների Կարտան-արտադրյալների $(ad)^n(X_2)^k$ քվանտային չափողականությունների համար:
- Կազիմիրի երկրորդ օպերատորի ունիվերսալ սեփական արժեքների ստացումը վերոնշյալ ներկայացումների վրա:
- Ունիվերսալ բանաձևերի՝ գծորեն լուծելիության հատկության սահմանումը: Ներկայում հայտնի ունիվերսալ քվանտային չափողականությունների՝ գծորեն լուծելիության հատկության ապացուցումը:

- Ունիվերսալ բանաձևերի երկրաչափական մեկնաբանումը: Ունիվերսալ չափողականությունների ոչ-միակության արտադրիչների ու երկրաչափական կոնֆիգուրացիաների միջև համապատասխանության հաստատումը: Չորս չափանի ոչ միակության արտադրիչի դուրսբերումը՝ օգտվելով $(16_3, 12_4)$ ոիրականացվող կոնֆիգուրացիայից:
- Կարտանի սիմետրիզացված մատրիցի դետերմինանտի արժեքի համար հայտնի Կաց-Պետերսոնի նույնության ընդհանրացումը: S^3 սֆերայի վրա սահմանված զտված Չերն-Սայմոնս տեսության վիճակագրական գումարի համար ճշգրիտ արտահայտության դուրսբերումը բոլոր պարզ խմբերի համար:
- Վերոնշյալ վիճակագրական գումարների համար ունիվերսալ տեսքի արտահայտությունների դուրսբերումը: Այս արտահայտությունները հնարավոր կդարձնեն ցանկացած պարզ խմբի համար զտված Չերն-Սայմոնս/տոպոլոգիական լարեր դուալությունների ուսումնասիրումը:

АВЕТИСЯН МАНЕ ЕРВАНДОВНА

УНИВЕРСАЛЬНОСТЬ ВОЖЕЛЯ И ЕЕ ПРИЛОЖЕНИЯ

Резюме

Идея универсальности Вожеля является эффективным альтернативным инструментом для изучения простых алгебр Ли и основанных на них физических теорий. В течение последних двадцати лет на ее основе изучалась и развивалась теория *универсальных* представлений простых алгебр Ли. Кроме того, сравнительно недавно выявился значимый вклад универсального подхода в вопросе изучения возможных дуальностей между разными физическими теориями.

В настоящей диссертации представлены результаты, касающиеся развития вышеуказанных направлений. Именно:

- вывод новой универсальной формулы для квантовых размерностей картановских произведений произвольных степеней присоединенного ($a\,d$) и X_2 представлений;
- вывод универсального выражения для собственных значений второго оператора Казимира на вышеупомянутых представлениях;
- определение свойства линейной разрешимости универсальных формул. Доказательство линейной разрешимости всех известных универсальных формул для квантовых размерностей;
- геометрическая интерпретация универсальных формул. Установление связи между *множителями неединственности* и геометрическими конфигурациями. Вывод четырехмерного множителя неединственности с помощью реализуемой конфигурации $(16_3, 12_4)$;
- обобщение тождества Каца-Петерсона для детерминанта симметризованой матрицы Картана. Вывод точного выражения для статистической суммы рафинированной теории Черна-Саймонса на S^3 для любой простой калибровочной группы;
- вывод выражений универсального вида для всех вышеуказанных статистических сумм. Эти выражения позволят изучить возможные

дуальности между теориями рафинированного Черна-Саймонсана для любой простой группы и теорией топологических струн.