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Դավթյան Մհեր Արայիկի

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Ա.04.03 «Ռադիոֆիզիկա» մասնագիտությամբ ֆիզիկամաթեմատիկական
գիտությունների թեկնածուի գիտական աստիճանի հայցման ատենախոսության

ՍԵՂՄԱԳԻՐ

ԵՐԵՎԱՆ - 2022

THE MINISTRY OF EDUCATION, SCIENCE, CULTURE AND SPORTS OF RA
YEREVAN STATE UNIVERSITY

Mher Davtyan

THE ROLE OF SYMMETRICAL INHOMOGENEITY IN PROBLEMS OF ELECTROMAGNETIC WAVE PROPAGATION

ABSTRACT

of the thesis for the degree of candidate of physical and mathematical sciences
Specialty 01.04.03 – “Radiophysics”

YEREVAN - 2022

Ատենախոսության թեման հաստատվել է Ռադիոֆիզիկայի և Էլեկտրոնիկայի ինստիտուտում:

Գիտական ղեկավար՝	ֆ.մ.գ.դ. Ժիրայր Սերգեյի Գևորգյան
Պաշտոնական ընդդիմախոսներ՝	ֆ.մ.գ.դ. Խաչատուր Վլադիմիրի Ներկարարյան ֆ.մ.գ.թ. Արմեն Էդվարդի Ալլահվերդյան
Առաջատար կազմակերպություն՝	Միջուկային հետազոտությունների միացյալ ինստիտուտ (ՄՀՄԻ), Դուբնա, Ռուսաստանի Դաշնություն

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The subject of the dissertation is approved by the scientific council of the Institute of Radiophysics and Electronics (IRPHE):

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Leading Organization:	Joint Institute for Nuclear Research, Dubna, Russian Federation

The defense will take place on 13th of July 2022 at 10:00 during the session of the 049 professional council of physics acting within Yerevan State University (1 Alek Manukyan str., 0025 - Yerevan)

The dissertation is available at the YSU library.

The abstract is sent out on 3rd of June.

Scientific secretary of the 049 council:
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Abstract

In this thesis we investigate various phenomena related to light propagation in media which possess different types of symmetric inhomogeneity. Our main focus is on two types of inhomogeneous media. Firstly, we examine photonic crystals which are optical structures where the refractive index changes periodically. Specifically, we examine the effect of resonant polarization rotation when the electromagnetic wave passes through a photonic crystal. We propose a theory based on Maxwell equations and the symmetric Kronig-Penney model which explains the abrupt polarization change and which is in good correspondence with the experiment. The other type of inhomogeneity that is examined is the case when the refractive index of a medium is a continuous function of position. Moreover, central focus is given to profiles possessing different additional symmetries. Specifically, we investigate the effects occurring in Maxwell Fish eye profile and the Luneburg profile. We have shown that that due to additional symmetry of the Maxwell Fish eye the expressions of the ray trajectories are derived using the integrals of motion. Other than that, we have also proposed a general mechanism which allows to find the deformation of any refractive profile which preserves the symmetries of the system and ensures that the ray trajectories remain closed when spin is taken into account. Finally, we also examine the radiation properties of Maxwell fish eye including radiation intensity, spectrum and the angular distribution of the radiation. We have found that there are three regimes of radiation when a charged particle passes through the MFE. A Cherenkov-like condition is also derived.

Relevance of the scientific research

Media with symmetric inhomogeneity have been of great interest due to numerous effects which occur when electromagnetic waves propagate through such media. In general, inhomogeneity of a certain medium means that the refraction index of the medium is not constant and has some sort of spatial dependency. These types of media are called gradient index (GRIN) media. Different optical effects related to GRIN media are described by GRIN optics. GRIN optics examines different phenomena related to the gradient of the refractive index of the medium and/or the material. It is known that one important advantage of GRIN materials is that those can be used to build optical systems for imaging without aberrations which are common for traditional optical systems like spherical homogenous lenses. GRIN materials are classified into three major categories – axial, cylindrical and spherical. In axial GRIN profiles the refractive index changes along the optical axis of the system. Cylindrical refractive profile is a system consisting of concentric cylinders with surfaces with constant refraction index. Spherical profiles, by analogy, are symmetric relative to point source. In spherical profiles surfaces of concentric spheres have constant refraction index. Electromagnetic waves propagating in such materials result in many effects which are being exploited to produce lenses, optical fibers and other devices. By symmetric inhomogeneity we refer to the case when the inhomogeneity of the medium or material is not an arbitrary function of position but has some sort of symmetry. In this dissertation our main focus is on two types of inhomogeneous media. Firstly, we examine photonic crystals which are optical structures where the refractive index changes periodically. The other type of inhomogeneity that is examined is the case when the refractive index of a medium is a continuous function of position. Moreover, central focus is given to profiles possessing different symmetries. Specifically, we investigate the effects occurring in Maxwell Fish eye profile which has spherical symmetry as well as an extended symmetry which will be thoroughly studied in Chapters 3 and 4. Photonic crystal is a system with periodical dielectric permittivity and has many interesting properties regarding transmission and polarization change of incident electromagnetic wave. Recent developments in material science have made possible the fabrication of photonic crystals that allow the observation of many peculiar effects [1], including perfect reflector for all polarizations over a wide selectable spectrum [2, 5], optical Hall effect [3], unidirectional scattering [4] with broken time reversal symmetry, the propagation of optical beams without spatial spreading and polarization rotation effects in dilute photonic crystal which were examined in [9]. In general, it is accepted that in geometrical optics approximation polarization change is negligible. Conversely, in the opposite limit, when the scale of characteristic inhomogeneity is of same order as that of the wavelength - rotation of polarization becomes much more noticeable. The problem of polarization change is particularly important because of its applications in polarization controlling devices and when there is a need to manipulate the polarization of incidence light [6–8]. Particularly, in the above-mentioned paper, it is experimentally observed that there appears a drastic change of polarization in photonic crystal which has inhomogeneity perpendicular to the propagation direction of the wave. In Section 2.2 we briefly present the experimental set-up that was used to detect the change of polarization. In the following two sections it is presented the developed theory which is based on Maxwell's equations with two-dimensional inhomogeneous permittivity. In [10] we have presented the theory for the TE waves and obtained a good correspondence between the theoretical and experimental frequencies for which the resonance polarization change was observed. Under certain conditions dilute photonic crystals are also argued to cause a light straightening effect [11]. It is found that such effect is caused by the formation of localized states in transversal motion. It is argued that these states decrease mobility in the transverse direction and force the light to be straightened. Starting from the second chapter we mainly focus on the other type of the inhomogeneity that was noted earlier. That is, we will examine various phenomena that arise when electromagnetic wave propagates through different kinds of continuous refractive media. Particularly, we have considered refractive profiles (Maxwell fish eye, Luneburg lens) possessing spherical symmetries which are of high scientific interest due to their applications in well-known phenomena of perfect imaging and cloaking. Cloaking phenomena have attracted a reasonable amount of interest since Pendry [12] and Leonhardt [13] assumed that an object coated by certain inhomogeneous shell becomes invisible to electromagnetic waves. Different mechanisms of cloaking have been suggested since that time, among which are anisotropic metamaterial shells [12], conformal mapping in two-dimensional systems [13], complementary media [14], etc. Transformation optics [12] is the most common approach, where the dielectric permittivity and magnetic permeability tensors are specific coordinate-dependent functions. However, this approach is difficult to implement in the optical field, since it is problematic to find metamaterials with the necessary magnetic properties [15]. In the cases when the photon wavelength is much smaller than the characteristic size of inhomogeneity, geometrical optics approximation is justified [16-19]. In the conformal mapping method of cloaking when geometrical optics approximation is used, closed ray trajectories are of significant importance [13]. The scheme that Leonhardt proposed uses optical conformal maps where a dielectric medium conformally maps physical space onto Riemann sheets. The idea is to send all rays that have passed through the branch cut onto the interior sheet back to the cut at precisely the same location and in the same direction in which they entered [13]. The latter illustrates that potentials (refractive profiles) which result to closed ray trajectories are particularly important in achieving the cloaking phenomenon. Moreover, these trajectories determine the size and shape of the cloaked area. Other than that, Maxwell's fish eye has also applications in quantum optics. While investigating quantum optical properties of MFE, it was shown that such a system mediates effectively infinite-range dipole-dipole interactions between atomic qubits, which can be used to entangle multiple pairs of distant qubits [21]. MFE was also used as a possible mean for spin waves focusing

[22]. Using numerical simulations, it was shown that sub-wavelength focusing can be achieved by means of MFE with appropriate chosen parameters. Furthermore, simulation results also prove that the focusing properties of MFE can be tuned by external magnetic field, so the MFE would be promising device for spin wave focusing and wavefront manipulation [22]. Maxwell fish eye potential energy profile was also considered in graphene quantum dot. It was shown that all the electron trajectories are closed circles that are classified by angular momentum and an additional integral of motion. Moreover, given that the lower Dirac zone is completely filled, a universal value for Hall conductivity is found [23]. Note that in this section of the dissertation light polarization is mostly neglected. In Chapter 4 we examine the propagation of polarized light in the medium with Maxwell fish eye refraction index profile. The chapter is organized as follows. In subsection 4.2, we describe the Hamiltonian formulation of the geometric optical system. We also present some other textbook facts on the duality between Coulomb and free-particle systems on a (pseudo)sphere which allow us to relate the Maxwell fish eye and Coulomb profiles and will be used in our further consideration. In subsection 4.2, we present the Hamiltonian formalism for the polarized light propagating in an optical medium and propose the general scheme of the deformation of an isotropic refraction index profile which allows us to restore the initial symmetries after the inclusion of polarization. In subsection 4.3, we use the proposed scheme for the construction of a “polarized Maxwell fish eye” profile which inherits all the symmetries of the original profile when light polarization is taken into account. We present the explicit expressions for the symmetry generators of the corresponding Hamiltonian system and find the expressions of the Casimirs of their symmetry algebra. In subsection 4.5 the explicit expressions for the trajectories of polarized light are presented. It is shown that these trajectories are no longer orthogonal to the angular momentum but turn to a fixed angle relative to it. Despite deviations from circles, these trajectories remain closed. We show that light polarization violates the additional symmetries of the medium so that ray trajectories no longer remain closed. Afterwards we suggest a modified, polarization-dependent Maxwell fish eye refraction profile which restores all the symmetries of the initial profile and yields closed trajectories of polarized light. Explicit expressions for the polarization-dependent integrals of motion and the solutions of corresponding ray trajectories are also presented. In Chapter 5 we consider problems related to light generation in mediums with different types of refraction index profiles. So far most of the attention was paid to light propagation in such systems. Other very interesting topic which also has a big potential in terms of real-life applications is light generation in such profiles. Within the scope of this dissertation, light generation mechanisms in a medium with highly symmetric Maxwell fish eye refractive index profile are investigated. Under the term light generation, we refer to radiation properties of the Maxwell fish eye when a charged particle interacts with the refraction profile by penetrating through it. In general, there are three main types of emitted radiation when a charged particle penetrates some medium. Those are Cherenkov radiation,

transition radiation and finally the so-called diffraction radiation. Radiation intensity and spectrum are primary objectives of investigation in any type of radiation problem. Apart from those, angular distribution of the radiation intensity is another important property which is examined in this section. Whether the radiation is isotropic or it is highly directed towards some direction. In this chapter radiation from a charged particle moving in a medium with Maxwell fish eye refraction index profile is considered. Maxwell's equations are used to describe the electromagnetic fields in an inhomogeneous medium taking into account the moving charge as an external source. Other than that, well-known method of Green's function is used to find out the expression for radiation potential A and corresponding electric and magnetic fields. Generally, Green's function is an integral kernel that can be used to solve many types of differential equations including inhomogeneous partial differential equations [20]. Also, we have constructed the well-known Poynting's vector which will be used to derive the radiation intensity. Whenever appropriate, numerical estimations of integrals are completed. We have shown that the radiation spectrum has a discrete character. The main emitted wavelength is proportional to the refractive profile's radius and has a dipole character in a regular medium. Cherenkov like threshold velocity is established. A cardinal rearrangement of angular distribution in a lossless medium is predicted. This behavior is caused by the total internal reflection in a lossless medium as opposed to photons' attenuated total reflection in the regular medium. Lossless medium ensures that both directed and monochromatic emission can serve as a light source in the corresponding regions.

Aim of the dissertation

The main objective of the dissertation is to investigate different phenomena related to light propagation in media with symmetric inhomogeneity. Main objectives of the thesis are the following:

- understand the polarization rotation mechanism in dilute photonic crystal
- investigate light propagation in Maxwell fish eye refraction profile focusing on the closed ray trajectories
- obtain the spin-induced deformation for an arbitrary refractive profile, which preserves the internal symmetries of the system
- understand light generation mechanisms by investigating the radiation properties of MFE

Results submitted for defense

- a new theory based on Maxwell equations has been proposed which explains the resonance polarization change in dilute photonic crystal
- parameters of ray trajectories in Maxwell fish eye have been expressed through the integrals of motion
- we observed the existence of a photon state with maximal angular momentum, which can be used as an optical resonator as well as the spin Hall effect in an extended symmetry profile has been predicted
- we have shown that polarization violates the additional symmetries of the medium so that ray trajectories no longer remain closed
- we suggest a modified, polarization-dependent Maxwell fish eye refraction index which restores all symmetries of the initial profile and yields closed trajectories of polarized light
- explicit expressions for the polarization-dependent integrals of motion and the solutions of corresponding ray trajectories are presented
- generalized scheme has been proposed which allows to find the deformation of an arbitrary refraction index profile when spin is taken into account
- the deformation of the well-known Luneburg profile is also presented
- it is shown that the radiation from a charged particle moving in a medium with a Maxwell's fish-eye refraction index profile has a discrete spectrum

Practical value

The practical value of the thesis is reflected in the fact that a number of phenomena, such as perfect imaging, the cloaking phenomenon etc. are closely related to the various properties of the MFE profile. The closed ray trajectory of the light beam determines the size and shape of the cloaked area. We have obtained spin-induced deformations of the well-known refraction index profiles (Maxwell fish eye, Luneburg), which preserve the symmetries of the system and thus the trajectories of the rays remain closed.

Other than that, we have proposed a general mechanism that allows to find the spin-induced deformation of any refraction profile. This mechanism can be used both in theoretical and experimental research.

When studying the trajectories of rays in the MFE profile, photon states with maximum angular momentum were observed. These states are very important in the sense that those can be used as optical resonators.

We also investigate the problem of radiation of a charged particle when it passes through the MFE profile. The results obtained are very important in the sense that they allow to obtain radiation in the microwave and terahertz bands. Since millimeter scale MFE profiles are practically possible, the obtained results can be used to generate radiation in the specified bands.

The novelty of the work

The presented research investigates various phenomena related to the propagation of light in media with symmetric inhomogeneity, focusing on the well-known refraction index profile Maxwell fish eye. We expand the current knowledge about the properties of the MFE. The spin-induced deformation of the MFE profile is one of the main novelties of the work. Moreover, the general scheme to obtain the spin-induced deformation of any profile is an important scientific novelty that is presented in this thesis. The other important novelty of this research is the investigation of the radiation properties of MFE.

Length and structure of the dissertation

The dissertation contains 6 chapters and the bibliography. The first and the last chapters are the Introduction and the Conclusion respectively. The other 4 chapters present our findings.

Content of the dissertation

Chapter 1

The first chapter of the dissertation is an introductory chapter where we present an overview about main concepts that are covered in the thesis. We present the two main types of media with symmetric inhomogeneity which are photonic crystals (periodic inhomogeneity) and the so called GRIN media, where the gradient index profile is some continuous function of coordinate. In addition, we illustrate the importance of our consideration in terms of applications in other well-known phenomena.

Chapter 2

In Chapter 2 the rotation of polarization of TE waves in one dimensional dilute photonic crystal is theoretically investigated. In Section 2.2 we describe the recent experimental results obtained in dilute photonic crystal where resonance polarization change is observed. In Section 2.3 we present the theory that was introduced in [9]. In Section 2.4 it is presented the reformulation of the same theory using TE waves. In the following sections it is examined the spectrum of the photonic crystal as well as comparison with the actual experimental results is completed. Resonance character of polarization rotation is revealed.

It is shown that rotation angle at resonant frequencies acquires only discrete values. Results of theory are compared with the experiment. Good correspondence between the TE theory and the experiment was observed.

Chapter 3

In this chapter we consider the refraction indices which yield the optical metrics coinciding with those of three-dimensional sphere (for positive κ) and two-sheet hyperboloid/pseudosphere (for negative κ),

$$n(\boldsymbol{r}) = \frac{\mathrm{n}_0}{|1+\kappa \boldsymbol{r}^2|}, \kappa = \pm\frac{1}{4r_0^2}.$$

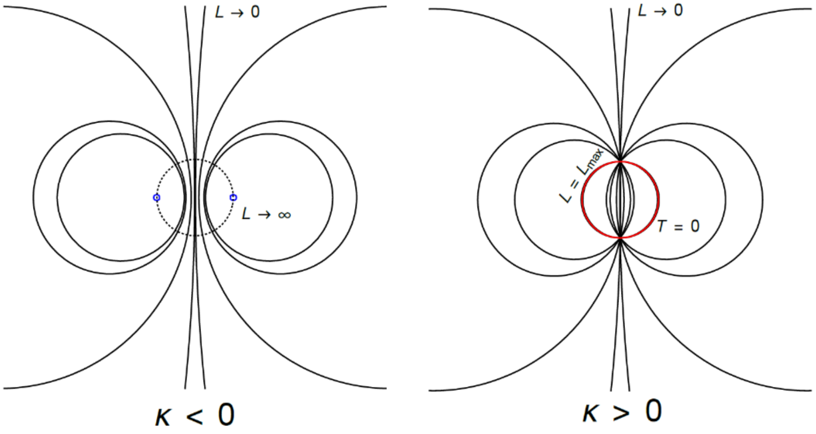

Figure 1. Ray trajectories in Maxwell fish eye.

While the generic homogeneous spaces have so(3) symmetry algebra generated by conserved angular momentum, three-dimensional sphere and pseudosphere have symmetry algebras so(4) and so(3.1), respectively. The reason for that is the existence of three additional conserved quantities - “translation generators”. This profile is well-known in optics as the “Maxwells fish eye”. Using the extended symmetries, we investigate the ray trajectories in this profile and aim to find the explicit expressions for ray trajectories. In Section 3.2 and 3.3 we present the Hamiltonian formulation of the system using the action of the system on the three-dimensional curved space equipped with the “optical metrics” (or Fermat metrics) of Euclidean signature $\mathrm{g}_{AB} = \mathrm{n}^2(\boldsymbol{r})\delta_{AB}$. Using the conserved quantities, we are able to find the explicit expressions for ray trajectories. Due to the so(4)/so(3.1) symmetry of three-dimensional sphere/hyperboloid, in addition to so(3) algebra generators $\boldsymbol{L} = \boldsymbol{r} \times \boldsymbol{p}$, the system possesses three more conserving quantities $\boldsymbol{T} = (1-\kappa\boldsymbol{r}^2)\boldsymbol{p} + 2\kappa(\boldsymbol{pr})\boldsymbol{r}$. Using these expressions, we are able to obtain explicit expressions for the ray trajectories. In Fig. 1 are depicted the ray trajectories for different values of $\boldsymbol{T}$, $\boldsymbol{L}$ and κ. Dashed and red circles with radius $2r_0$ are given by the refraction profile $n(\boldsymbol{r})$. The red circle is the photon trajectory with maximal angular momentum of the photon in $\kappa > 0$ case. The straight lines are trajectories without angular momentum $L = 0$. The small blue circles are trajectories in $L \to \infty$ limit provided that $\kappa < 0$. The main results obtained in this chapter are the following:

- the ray trajectories in Maxwell fish eye are closed and the expressions for ray trajectories are found directly from the integrals of motion
- there exists a photon state with maximal angular momentum and it can be used as a possible means for creating optical resonators
- a perturbation theory has been proposed which illustrates that when we take into account the spin the trajectories rotate by a fixed angle

Chapter 4

In this chapter we continue our study of a polarized light passing through the Maxwell fish eye profile within the geometrical optics approximation. The key point of our study is that we introduce a polarization-dependent deformation of the Maxwell fish-eye profile

$$\mathrm{n}_{\mathrm{Mfe}}^{\mathrm{s}}(\boldsymbol{r}) = \frac{\mathrm{n}_{\mathrm{Mfe}}(\boldsymbol{r})}{2}\left(1 + \sqrt{1 - \frac{4\kappa \mathrm{s}^2 \lambdabar_0^2}{\mathrm{n}_0}\frac{1}{\mathrm{n}_{\mathrm{Mfe}}(\boldsymbol{r})}}\right),$$

where $\mathrm{n}_{\mathrm{Mfe}}^{\mathrm{s}}(\boldsymbol{r})$ is the original Maxwell fish eye profile, and s is the light polarization. For the linearly and circularly polarized light we have $\mathrm{s} = 0$ and $\mathrm{s} = 1$ respectively. The proposed deformation restores all the symmetries of the optical Hamiltonian, with Maxwell fish eye profile, which were broken after the inclusion of polarization. It also ensures the closeness of the trajectories for polarized photons and can be used for cloaking and perfect imaging of polarized photons. The integrals of motion when light polarization is taken into account are the following:

$$\mathbf{J} \to \mathbf{r} \times \mathbf{p} + \mathrm{s}\frac{\mathbf{p}}{\mathrm{p}}\,, \quad \boldsymbol{T_s} = \left(2 - \frac{\mathrm{n}_0}{\mathrm{n}_{\mathrm{Mfe}}^{\mathrm{s}}(r)}\right)\boldsymbol{p} + 2\kappa(rp)\boldsymbol{r} + \frac{2\kappa \mathrm{s}}{\mathrm{n}_{\mathrm{Mfe}}^{\mathrm{s}}(\mathrm{r})}\boldsymbol{J}.$$

It is seen, that spin induced term is proportional to the dimensionless parameter $\mathrm{s}^2\lambdabar_0^2/r_0^2$ where r_0 is the characteristic length of the profile $n(\boldsymbol{r})$. This means that spin will play a significant role only in the vicinity of wave and geometrical optics border $\mathrm{s}\lambdabar_0/r_0 \sim 1$, since we are working in the framework of geometrical optics approximation, $\lambdabar_0 \ll r_0$.

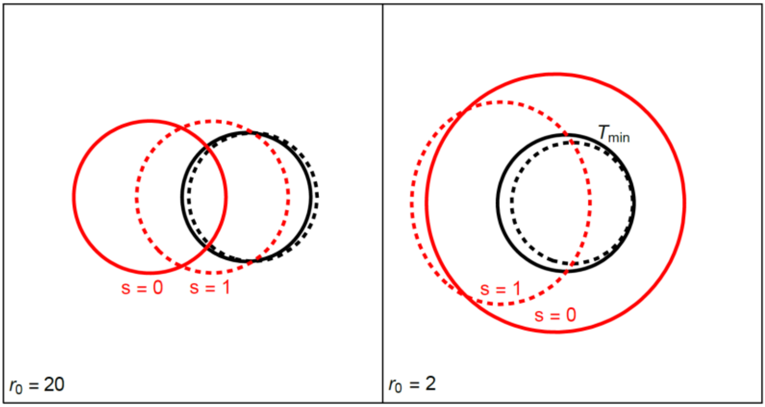

Figure 2. Deformations of the ray trajectories.

In Fig. 2 are illustrated the deformations of the ray trajectories for different values of r_0 when $n_0 = 1.5$, $\lambda_0 = 1$. The black curves correspond to the basic trajectories where $T = T_{\min} = s/r_0$. The red (light gray) curves correspond to trajectories with intermediary value of T. Dashed curves are the trajectories corresponding to the same value of T but for circularly polarized light (s = 1). The first figure ($r_0 = 20$) corresponds to the case when the deformations of the profile only result to the shift of the centers of the trajectories not affecting their shapes. Conversely, in the second figure ($r_0 = 2$), the deformations of the Maxwell fish eye result to highly deformed trajectories.

Chapter 5

As opposed to the research regarding light propagation in Maxwell fish eye medium, much less attention has been paid to light generation problems in that particular medium. However, it turns out that radiation emitted by a charged particle when it passes through such a medium possesses unique properties as well (see below). In this chapter, we consider the spectrum and angular distribution of radiation from a charged particle moving in a Maxwell fish eye refraction profile medium. Instead of dyadic Green's function, we utilize the exact Green's function of scalar Helmholtz equation [24-26]. This approach allows us to obtain complete analytical expressions for radiation intensity that reveal new physical results. The geometry of the problem is presented in Fig. 3.

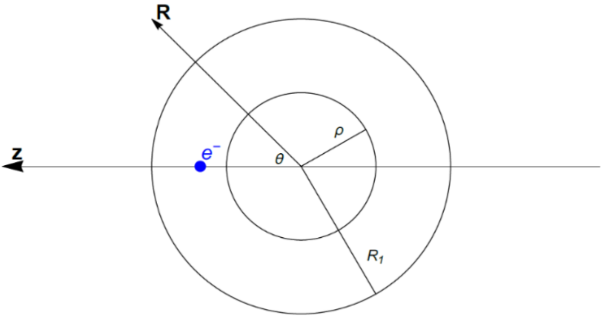

Figure 3. Geometry of the problem. Observation point is far away from the charge and from the core of the refraction profile.

Starting from the Maxwell's equations and after expressing vectors $\boldsymbol{E}$ and $\boldsymbol{B}$ in terms of the vector potential we arrive at the following equation: $\nabla^2\mathbf{A} + \frac{\omega^2}{c^2}\varepsilon(\mathbf{r})\mu(\mathbf{r})\mathbf{A} = -\frac{4\pi}{c}\mu\mathbf{j}$**. The** gauge condition for an inhomogeneous medium is given by: $\nabla(\nabla \cdot \boldsymbol{A}) + \frac{i\omega}{c}\varepsilon(\boldsymbol{r})\mu(\boldsymbol{r})\nabla\phi = 0$, where is the scalar potential and is determined as $\boldsymbol{\nabla}\phi = \boldsymbol{E} - \frac{i\omega}{c}\boldsymbol{A}$. The radiation potential can be expressed through Green's function of the scalar Helmholtz equation: $A_{zr}(\boldsymbol{R}) = -\frac{4\pi}{c}\int d\boldsymbol{r} G(\boldsymbol{R},\boldsymbol{r})\mu(\boldsymbol{r})\boldsymbol{j}_z(\boldsymbol{r})$. Green's function satisfies the equation: $\left[\boldsymbol{\nabla}^2 + \frac{\omega^2}{c^2}n^2(\boldsymbol{R})\right] G(\boldsymbol{R},\boldsymbol{r}) = \delta(\boldsymbol{R}-\boldsymbol{r})$, where $n(\boldsymbol{r}) = \sqrt{\varepsilon(\boldsymbol{r})\mu(\boldsymbol{r})}$, where $n(r) = \frac{2n_0\rho^2}{r^2+\rho^2}, \quad r < R_1$ and $n(r) = 1$ when $r > R_1$. Green's problem is exactly solved and has singularities expressed by the parameter ν, which has the following form

$$\nu = \frac{-1 + \sqrt{1 + 4n_0^2k^2\rho^2}}{2}$$

When $\nu = n + 1/2, -n - 3/2$ $(n \in N)$ Green's function is divergent and there is another expression which takes into account those singularities. Fourier component of the current density along the $\boldsymbol{z}$ axis has the following form: $\boldsymbol{j}(\boldsymbol{r},\omega) = \frac{e\boldsymbol{v}}{v}\delta(x)\delta(y)e^{ik_0z}$, where $k_0 = \omega/v$. Assuming a small imaginary part for n_0 and correspondingly for ν at the singular points the integral for vector potential is calculated analytically:

$$A_{1/2}(R) = -\frac{4e}{c}\frac{i\,\mathrm{sign}(v)K_0(k_0\rho)}{\sinh\left(\pi\mathrm{Im}\left[\nu_{1/2}\right]\right)}\frac{\rho}{\sqrt{R^2+\rho^2}}$$

$$A_{3/2}(R) = -\frac{8ie\,\mathrm{sign}(v)}{c\sinh\left(\pi\mathrm{Im}\left[\nu_{3/2}\right]\right)}\frac{\rho}{\sqrt{R^2+\rho^2}}\left[(k_0\rho)K_0(k_0\rho) - K_1(k_0\rho)\right], \quad R < R_1$$

In the absence of external sources, the isotropic solution when $R > R_1$ is chosen in the form: $A_v(R) = C\frac{e^{ikR}}{R}, \quad R > R_1$. We find the constant *C* from the boundary condition and arrive to following equation: $C_{1/2}(R_1) = -\frac{4ie\mathrm{sign}(v)}{c\sinh(\pi \mathrm{Im}\nu_{1/2})}\rho K_0(k_0\rho)e^{-ikR_1}$. Using the explicit expressions of magnetic field components, we can calculate the electromagnetic field energy density and thus the radiation intensity:

$$I_{1/2}(\theta) = \frac{3e^2}{\pi c}\frac{K_0^2\left(\frac{\sqrt{3}}{2\beta\sqrt{\varepsilon}}\right)}{\sinh^2\left(\frac{3\pi \mathrm{Im}[\varepsilon]}{8\varepsilon}\right)}\sin^2\theta,$$

where $\beta = v/c$. Since the modified Bessel function is exponentially small for large values of the argument, for the existence of radiation, the condition $\beta > \frac{\sqrt{3}}{2\sqrt{\varepsilon}}$ should be satisfied. In Fig.4 we can see the *R* dependence of radiation potential for different angles. The radiation potential in the lossless medium is highly anisotropic. It follows from Fig. 4 that at large observation angles the potential is significantly smaller. The maximum potential is reached at small angles from the particle trajectory. Modifying the imaginary part of n_0, one can observe a transition from highly directed to isotropic radiation potential. The isotropic radiation potential leads to a dipole-like radiation intensity. However, in the lossless medium the radiation potential is highly directed. Note that the *R* dependence for all angles is ~1/*R* (see Fig. 4). The *U*-shaped curve is obtained for the lossless medium using generalized Green's function. The actual radiation potential is determined by the curve below the straight lines.

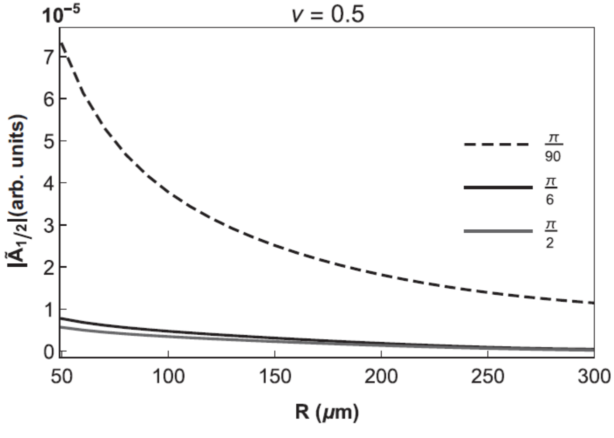

Figure 4. R dependence of radiation potential for different angles.

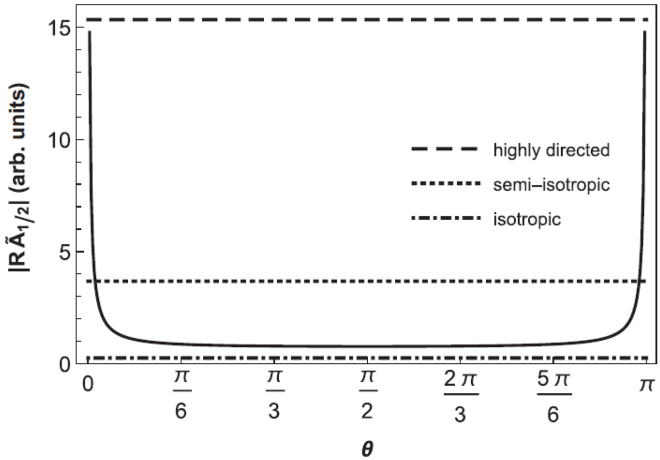

Figure 5. Angular distribution of radiation vector potential.

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ՀԱՄԱՉԱՓ ԱՆՀԱՄԱՍԵՌՈՒԹՅՈՒՆՆԵՐ ԴԵՐԸ ԷԼԵԿՏՐԱՄԱԳՆԻՍԱԿԱՆ ԱԼԻՔՆԵՐԻ ՏԱՐԱԾՄԱՆ ԽՆԴԻՐՆԵՐՈՒՄ

Ամփոփում

Աշխատանքում ուսումնասիրվել են տարատեսակ համաչափ անհամասեռություններով օժտված միջավայրերում լույսի տարածման հետ կապված խնդիրներ: Միջավայրի անհամասեռությունը նշանակում է, որ միջավայրի բեկման ցուցիչը հաստատուն չէ և ունի որոշակի տարածական կախվածություն: Այս ատենախոսության մեջ մեր հիմնական ուշադրությունը կենտրոնացած է երկու տեսակի անհամասեռ միջավայրերի վրա: Նախ ուսումնասիրվում են ֆոտոնային բյուրեղները, որոնք օպտիկական կառուցվածքներ են օժտված պարբերական անհամասեռություններով: Անհամասեռության մյուս տեսակը այն է, երբ միջավայրի բեկման ցուցիչը որոշակի անընդհատ ֆունկցիա է կոորդինատից: Հիմնական ուշադրությունը տրվում է տարբեր ընդլայնված համաչափություններ ունեցող պրոֆիլներին: Մասնավորապես, ուսումնասիրված են «Մաքսվելի ձկան աչք» (ՄՁԱ) կոչվող պրոֆիլում լույսի տարածման և նույն պրոֆիլում ճառագայման հետ կապված խնդիրներ: Թեկնածուական ատենախոսության մեջ ստացված հիմնական արդյունքերը հետևյալն են՝

- Առաջ է քաշվել Մաքսվելի հավասարումների վրա հիմնված տեսություն, որը բացատրում է ֆոտոնային բյուրեղում բևեռացման ռեզոնանսային պտույտը:
- ՄՁԱ պրոֆիլում լույսի ճառագայթների հետագծերի պարամետրերը արտահայտվել են շարժման ինտեգրալների միջոցով:
- Բացահայտվել են ֆոտոնի այնպիսի վիճակներ, որոնք օժտված են առավելագույն պտտական մոմենտով և որոնք կարող են օգտագործվել օպտիկական ռեզոնատորներ ստանալու համար:
- Ստացվել է սպինից կախված ՄՁԱ պրոֆիլի փոփոխված արտահայտություն, որը վերականգնում է համակարգի ներքին համաչափությունները և հանգեցնում է բևեռացված լույսի ճառագայթների փակ հետագծերի:
- Գտնվել են սպինից կախված շարժման ինտեգրալների, ինչպես նաև բևեռացված լույսի ճառագայթների հետագծերի ճշգրիտ արտահայտությունները:
- Ստացվել է ունիվերսալ մեխանիզմ, որը թույլ է տալիս գտնել ցանկացած ռեֆրակցիոն պրոֆիլի ընդհանրացումը այն դեպքում, երբ հաշվի է առնվում սպինը:

- Ներկայացվել է Լունեբուրգի պրոֆիլի արտահայտությունը հաշվի առնելով սպինը:
- Ցույց է տրվել, որ ՄՁԱ բեկման ցուցիչով միջավայրով անցնող լիցքավորված մասնիկի ճառագայթման սպեկտրը դիսկրետ է:
- Ցույց է տրվել, որ ճառագայթվող ալիքի երկարությունը ուղիղ համեմատական է ՄՁԱ պրոֆիլի շառավղին և սովորական միջավայրում ունի դիպոլային ճառագայթման բնույթ:
- ՄՁԱ պրոֆիլում ճառագայթում ունենալու համար ստացվել է նաև Չերենկովի պայմանին համանման պայման ՄՁԱ պրոֆիլի համար:

РОЛЬ СИММЕТРИЧНЫХ НЕОДНОРОДНОСТЕЙ В ПРОБЛЕМАХ РАСПРОСТРАНЕНИЯ ЭЛЕКТРОМАГНИТНЫХ ВОЛН

Резюме

В данной диссертации исследуются явления, связанные с распространением света в средах, обладающих различными типами симметричной неоднородности. Неоднородность конкретной среды означает, что показатель преломления непостоянен и имеет определенную пространственную зависимость. Большей частью, внимание уделяется двум типам неоднородных сред. Во-первых, изучаются фотонные кристаллы, представляющие собой оптические структуры с периодическими неоднородностями. Другой тип — это когда показатель преломления является непрерывной функцией координаты. Основное внимание уделяется профилям с различными расширенными симметриями. В частности, изучались проблемы, связанные с распространением света в профиле, называемом «Рыбий глаз Максвелла» (МДА), а также проблема излучения в этом же профиле.

Основные результаты диссертации представлены ниже:

- Выдвигается теория, основанная на уравнениях Максвелла, объясняющая резонансное вращение поляризации в фотонном кристалле.
- Показано, что в профиле РГМ параметры траекторий световых лучей выражаются интегралами движения.
- Определены фотонные состояния с максимальным вращательным моментом, которые можно использовать для получения оптических резонаторов.
- Получено модифицированное выражение спин-зависимого профиля РГМ, которое восстанавливает внутренние симметрии системы и приводит к замкнутости траекторий поляризованных световых лучей.
- Найдены точные выражения интегралов движения, а также выражения траекторий поляризованных световых лучей.
- Получен универсальный механизм, позволяющий обобщить любой рефрактивный профиль с учетом спина.
- Представлено выражение профиля Люнебурга с учетом спина.
- Показано, что спектр излучения заряженной частицы, проходящей через среду с показателем преломления РГМ, является дискретным.
- Показано, что длина волны излучаемой волны пропорциональна радиусу профиля РГМ и в нормальной среде имеет характер дипольного излучения.

- Аналогично черенковскому условию, получено условие для излучения в профиле МДА.