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Report on the dissertation by Gevorg Mnatsakanyan titled 'Estimates of sparse and Carleson type operators'

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The dissertation by Gevorg Mnatsakanyan, Estimates of sparse and Carleson type operators, is cumulative, based on three publications.

The first publication is "On almost-everywhere convergence of Malmquist-Takenaka series", which appeared in the prestigious "Journal of Functional Analysis". The second and third publication are two papers on sparse operators in the Journal of Contemporary Mathematical Analysis, "On a weak type estimate for sparse operators of strong type" with G. Karagulyan and "Sharp weighted estimates for strong-sparse operators" as single author.

I am most familar with the first paper on Malmquist-Takenaka series, and for my own convenience will put most emphasis on this paper in my evaluation. This is an outstanding original paper, which by itself deserves a PhD by any international standards on quality and depth. It shows both technical mastery of a difficult technique as well as some very insightful original observations on a beautiful subject.

The Malmquist-Takenaka series can be viewed as a perturbation of Taylor- or Fourier series of a holomorphic function (say complex Hardy space H^p) in the unit disc. One can obtain the Taylor series of such function at the origin by the following iterative procedure. One subtracts a constant from the function so that the new function vanishes at the origin, then one divides the function by the argument z so as to obtain a new holomorphic function in the disc. Then one iterates. The sequence of of constants that one subtracts are the Taylor coefficients of the function at the origin, or equivalently, the Fourier coefficients of the function interpreted as function on the unit circle.

The Malmquist Takenaka series arises as the following perturbation. One chooses a sequence of numbers a_n in the open unit disc and applies a similar procedure as above where at the n-the step one replaces the role of the origin by a_n, that is, one subtracts a constant so that the new function vanishes at a_n and then divides by the Blaschke factor which vanishes at a_n. Clearly, if all a_n are zero, the Malmquist-Takenaka series specializes to the Fourier series.

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Interest in Malmquist Takenaka series arises in engineering, where one likes to choose the sequence a_n adaptive to the function that one tries to expand. For example, many of the numbers a_n are already places where the function vanishes. Such algorithms are then called non-linear phase unwinding. Numerical experiments show that in practice nonlinear phase unwinding converges super fast to the function. However, making this fast convergence mathematically rigorous remains a challenging and interesting open problem.

The focus in this thesis is to freeze the sequence a_n, thus make this sequence independent of the function as in the classical Malmquist Takenaka series, and ask pointwise almost everywhere convergence of the sequence for H^p functions. Even in the case of Fourier series, such convergence is non-trivial and tantamount to the classical result by Carleson.

Gevorg proves that if the sequence a_n lies in a compact set inside the open unit disc, then the Malmquist Takenaka series converges almost everywhere on the unit circle for functions in H^p, $1 < p < \infty$. This theorem is strictly more general than the classical Carleson-Hunt theorem about convergence of Fourier series. Indeed, it uses the technique of the even more difficult so-called polynomial Carleson theorem, an original result by Victor Lie, and in more general setting proven by Pavel Zorin-Kranich. The original idea by Gevorg is that the various phase functions arising from Blaschke factors form a family of functions with similar compactness properties as polynomials, provided the zeros of the Blaschke factors lie in a compact set. This is a result of great strength, using very deep arguments, following the technical tour de force by Zorin-Kranich but also inserting new ideas at various places. I would say this is the first non-trivial extension of the work of Zorin-Kranich with a very interesting application.

Gevorg then discusses in detail the compactness assumption for the zeros. He proves that the bounds for the maximal operator blow up as the compact set approaches the unit disc. This maybe unfortunate blow-up came as a surprise to me, certainly the original hope was to prove boundedness uniformly. Gevorg's example is ingeniously set up and a very nice argument.

On the other hand, if the zeroes lie in a non-tangential approach region, then one again has boundedness of the maximal partial sum operator.

In summary, this chapter contains a numebr of interesting observations on almost everywhere convergence of Malmquist Takenaka series. It contains various novel result with quite different ideas involved. I find this a very nice piece of work, surely satisfying very satisfactorily the requirements for a dissertation at highest international standards.

I will say a few words on the chapter on sparse operators. Sparse operators are presently a topic of widespread interest. This chapter contains several observations on a very general class of sparse operators. Clearly these are very good results, quite different in nature of the chapter on Malmquist-Takenaka series. These results are very good and give great confidence that Gevorg is capable on state-of-the-art

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research in more than one particular area.

Overall, this is an excellent thesis. I recommend it be accepted with highest possible grade.

Sincerely,

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