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(ԵՐԵՎԱՆԻ ՖԻԶԻԿԱՅԻ ԻՆՍՏԻՏՈՒՏ)

Խլղաթյան Շանթ Գողի

Լենզե-Թիրինգի երևույթը և մոդիֆիկացված գրավիտացիա

Ա.04.02 - «Տեսական ֆիզիկա» մասնագիտությամբ ֆիզիկամաթեմատիկական  
գիտությունների թեկնածուի գիտական աստիճանի հայցման ատենախոսության

ՍԵՂՄԱԳԻՐ

ԵՐԵՎԱՆ – 2023

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A.I. ALIKHANYAN NATIONAL SCIENCE LABORATORY  
(YEREVAN PHYSICS INSTITUTE)

Shant Khlghatyan

Lense-Thirring effect and modified gravity

SYNOPSIS

of Dissertation in 01.04.02 – Theoretical physics presented for the degree of candidate  
in physical and mathematical sciences

YEREVAN – 2023

Ատենախոսության թեման հաստատվել է Ա. Ի. Ալիխանյանի անվան Ազգային Գիտական Լաբորատորիայի (ԵրՖԻ) գիտական խորհուրդում:

Գիտական ղեկավար՝

Ֆիզ.մաթ. գիտ. դոկտոր

Գուրզադյան Վահագն Գրիգորի (ԱԱԳԼ)

Պաշտոնական ընդդիմախոսներ՝

Ֆիզմաթ. գիտ. դոկտոր,

Սեդրակյան Արա Գրիգորի (ԱԱԳԼ)

Ֆիզմաթ. գիտ. թեկնածու

Սամսոնյան Մարինե Մանվելի (ԱԱԳԼ)

Առաջատար կազմակերպություն՝

Ֆիզիկայի և աստղաֆիզիկայի բաժին, Միսսուրի համալսարան, ԱՄՆ

Ատենախոսության պաշտպանությունը կայանալու է 2023 թ. ապրիլի 19-ին ժամը 14:00-ին, ԱԱԳԼ-ում գործող ԲՈԿ-ի 024 «Ֆիզիկայի» մասնագիտական խորհրդում (Երևան, 0036, Ալիխանյան Եղբայրների փ. 2):

Ատենախոսությանը կարելի է ծանոթանալ ԱԱԳԼ-ի գրադարանում:

Սեղմագիրն առաքված է 2023 թ. մարտի 9-ին:

Մասնագիտական խորհրդի գիտական քարտուղար՝

Ֆիզ.մաթ գիտ. դոկտոր

Հրաչյա Մարուքյան

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The subject of the dissertation is approved by the scientific council of the A.I. Alikhanyan National Science Laboratory (YerPhI).

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The defense will take place on the 19<sup>th</sup> of April 2023 at 14:00 during the "Physics" professional council's session of SCC 024 acting within AANL (2 Alikhanyan Brothers str., 0036, Yerevan).

The dissertation is available at the AANL library.

The synopsis is sent out on the 9<sup>th</sup> of March, 2023.

Scientific secretary of the special council:

Doctor of ph-math. sciences

Hrachya Marukyan

## **Abstract**

The dissertation is devoted to the study the Lense-Thirring (LT) effect in a weak-field modification of General Relativity, also are considered other associated phenomena and effects. The extension of Newton's shell theorem (sphere-point gravity equivalency) and and the appearance of the cosmological constant in McCrea-Milne cosmological scheme, enable to study a bunch of observable effects in the relevant modified gravity such as the dynamics of groups and clusters of galaxies, galactic halos, the LT precession, shadows of black Holes (BH), etc.

Namely, based on the aforementioned modification of weak field General Relativity, the frame dragging precession is studied in Kerr metric, the frequency and other relevant formulae are derived for certain type of orbits, then satellite data on LT effect are used to constrain the modified gravity contribution. In Appearance of an additional term associated to the cosmological constant and LT effect is shown within the framework of gravito-electromagnetism.

Then the immediate environment of a black hole, is studied, that is the shadow and the accretion disk. Using the derived expression for the radius of the shadow of the Schwarzschild-de Sitter black hole, the observational data for M87\* supermassive black hole are studied and relevant constraints are obtained within the parameterized post-newtonian formalism (PPN). Expressions for the Schwarzschild-de Sitter metric for adiabatically invariant quantities - action variables are derived, in terms of the Weierstrass elliptic  $\wp$  function and relevant consequences and applications are analysed. The stellar tidal disruption effect near the massive black holes is studied, relevant formulae are derived for the case of modified gravity, including the star flux to the loss-cone.

## **Timeliness and relevance**

Various modified theories of gravity are currently used to study the nature of the dark matter and dark energy. The studied weak field General Relativity is among those approaches and enables one the comparison of its consequences with observational predictions for various phenomena. Among the latter are the frame dragging (Lense-

Thirring) effect, the black hole shadow, accretion disks, stellar tidal disruption near massive black holes, etc.

## **Aim of the dissertation**

- Analysis of a bunch of physical effects within the weak field modified General Relativity and comparison of the predictions with available observational data.

## **Novelty of the work**

The contribution of the cosmological constant is revealed in a number of astrophysical phenomena, in the strong field frame dragging, black hole shadow etc, the relevant formulae are derived and constraints are obtained in each case.

- In the strong field limit, i.e. in the Kerr metric, it was shown that the Lense-Thirring effect (which derived in the weak field limit) continues to be valid, and the maximum possible deviation from it is well described by the nutation of the angular momentum.
- Formulas were derived taking into account the cosmological constant both in a weak field (Lense-Thirring metric) and in the case of a strong field (Kerr metric). These expressions can be used to analyze observational data. In addition, an interpretation was given of the additional term arising in the precession rate.
- It is shown that in the case of cosmological constant modified gravity, the orbital eccentricity ceases to be an adiabatically conserved quantity, in contrast to the Keplerian case.
- it was shown that if the influence of the cosmological constant on the process of tidal destruction by a supermassive black hole is taken into account, the stellar flux increases. In addition, it is shown that pulsars can cross the horizon of a supermassive black hole without being destroyed by tidal forces, which makes it possible to study the geometry of the immediate vicinity of a black hole.

## **Practical value**

With the improvement of the accuracy of the observational data, the derived relationships for various astrophysical phenomena can be used to reveal the contribution of departures from the General Relativity.

## **Main points to defend**

- The consequences of weak field General Relativity with cosmological constant are studied for various astrophysical phenomena.
- The appearance of additional term with cosmological constant is shown for Lense-Thirring effect within gravito-electromagnetic formalism. The relevant formulae are derived regarding the physical effects near the immediate vicinity of a massive black hole, i.e. the shadows and the accretion disks. Using the observational data for the shadow of the supermassive black hole in galaxy M87, the relevant constraints on the parameter modification are obtained. For disk accretion the modified apsidal and nodal frequencies are obtained.
- Expressions for the adiabatically invariant action quantities for the Schwarzschild-de Sitter metric are derived, with relevant consequences and applications.
- The modification of the parameters, particularly of the stellar flux to the loss cone are derived for stellar tidal disruption effect near massive black holes in galactic nuclei.

## **Structure of the dissertation**

The dissertation consists of Introduction, three chapters:

1. Lense–Thirring Effect
2. Black Hole Shadow and Disk Accretion
3. Adiabatic Invariance and Tidal Disruption Effects

conclusion, list of used literature, includes figures and tables.

## Content of the dissertation

In the Introduction the role of modified gravity theories is discussed in the context of the problem of dark energy and dark matter. The Lense-Thirring effect and other observational effects are analysed which can be relevant for testing the modified gravity theories. The content of the chapters of the dissertation is given.

### Chapter 1

In the first chapter the Lense-Thirring precession in the Kerr metric is considered, the evolution of the angular momentum of a particle for both bounded and unbounded orbits, and an additional effect, namely, nutation of the angular momentum is

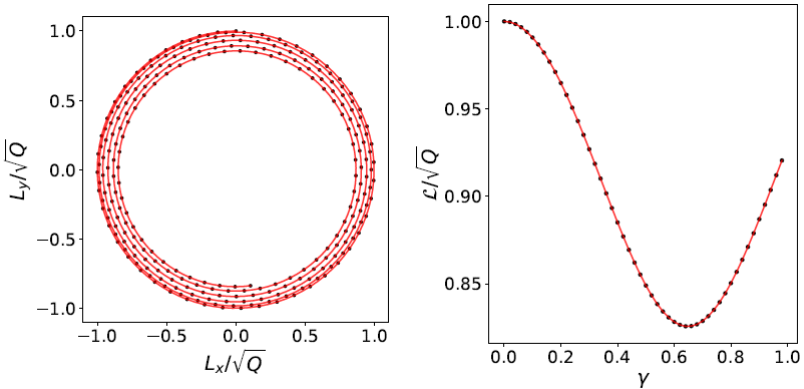


Figure.1 Left: A hodograph of a bounded orbit approximated by a sum of precession and nutation. Right: The squared radius in  $(L_x;L_y)$ -plane as a function of integration parameter . Dots depict the exact curves obtained through numeric simulation by the classical fourth-order Runge–Kutta method with absolute numeric error estimated to be  $< 10^{-7}$ , according to Runge’s rule. Red line shows the approximation.

revealed. The results are confirmed by means of numerical integration of geodesic equations (Fig.1 for a bounded orbit Fig.2 for a unbounded).

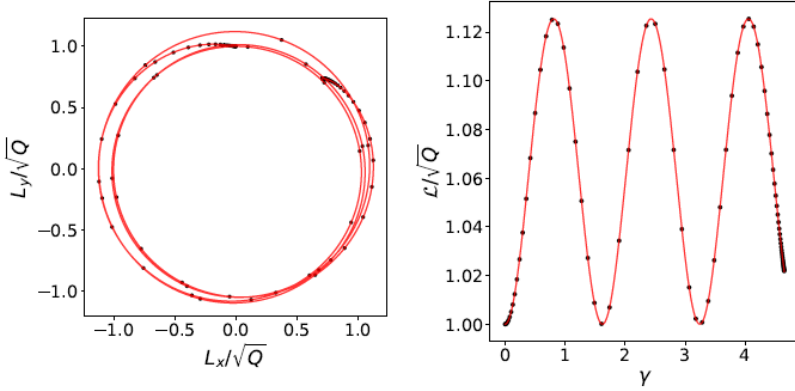


Figure 2. Left: A hodograph of an unbounded orbit approximated by a sum of precession and nutation. Right: The squared radius in  $(L_x; L_y)$ -plane as a function of integration parameter . Dots depict the exact curves obtained through numeric simulation by a variable step fourth-order Runge–Kutta method with absolute numeric error  $< 10^{-10}$ . Red line shows the approximation

In the same chapter the contribution of the cosmological constant is studied both in the weak field limit, i.e. in the Lense-Thirring metric, and in a strong field, in the Kerr metric. Expressions for precession rate in both weak field and strong field regimes are given in the following equations:

$$\Omega_{tot} = \Omega_{GR} + \Omega_{\Lambda} = \frac{2GJ}{c^2 a^3 (1 - e^2)^{3/2}} + \frac{\Lambda J}{3M}$$

$$\Omega = - \frac{\frac{ac2GMr}{c^2} + \frac{c\Lambda}{3}(ar^4 + a^3r^2 + a^3r^2 \cos^2 \theta + a^5 \cos^2 \theta)}{a^2 \sin^2 \theta (r^2 + a^2 - \frac{\Lambda}{3}(r^4 + a^2r^2)) - \frac{2GMr}{c^2}} - (r^4 + a^4 + 2a^2r^2) \left(1 + \frac{a^2\Lambda \cos^2 \theta}{3}\right)$$

It is shown the impossibility of interpretation of the cosmological constant an exotic matter of density  $\frac{-\Lambda c^2}{4\pi G}$ . Since without mass, the cosmological constant by itself cannot cause precession (as seen from the above expression).

The key to the interpretation of the additional term is given by the so-called electrogravitomagnetism. In this formalism, the Lense-Thirring precession (a purely relativistic gravitational effect) is put in correspondence with the Larmor precession (electromagnetic effect).

Within the framework of the gravito-electromagnetic formalism the Lense-Thirring precession is analysed for the modified gravity case and the precession rate obtained as a correction in the gyromagnetic ratio

$$\Omega = \frac{2GJ}{c^2 r^3} + \frac{\Lambda J}{3M}$$

$$\gamma = 2 \left( 1 + \frac{M_\Lambda}{M} \right)$$

In addition, limits on cosmological constant are obtained using the derived relations for the precession rate and The data from the LAGEOS-1, LAGEOS-2 and LARES satellites are used to obtain constraints on the modified gravity (Table 1).

Error limits of $\Lambda$ for LT effect				
Satellite	accuracy	semi major axis (m)	eccentricity	$\Lambda (m^{-2}) <$
LAGEOS-1	0.200	12271150.0	0.004456	$2.874459 \times 10^{-24}$
LAGEOS-2	0.200	12161840.0	0.013730	$2.953411 \times 10^{-24}$
LARES	0.050	7822000.0	0.000800	$2.774508 \times 10^{-24}$
LARES 2	0.002	12270000.0	0.002500	$2.875208 \times 10^{-26}$

Table 1



In the more realistic case, i.e when considering the oblateness of the earth we can use the contribution of the high tidal modes

$$\Lambda \leq \mathbb{E}(\Omega) \frac{3GM^2 R^2 \omega}{5J_c^2 r^3} \left[ 1 - \frac{219}{392} J_2 \left( \frac{R}{r} \right)^2 \right],$$

where  $J_2$  is the Bessel function of the first kind and we get more stringent constraints on the value of the cosmological constant

Error limits of $\Lambda$ for LT effect in more realistic case				
Satellite	accuracy	semi major axis (m)	eccentricity	$\Lambda (m^{-2}) <$
LAGEOS-1	0.200	12271150.0	0.004456	$1.881532 \times 10^{-31}$
LAGEOS-2	0.200	12161840.0	0.013730	$1.932717 \times 10^{-31}$
LARES	0.050	7822000.0	0.000800	$1.815726 \times 10^{-31}$
LARES 2	0.002	12270000.0	0.002500	$1.882061 \times 10^{-33}$

Table 2

## Chapter 2

In the second chapter the processes occurring near black holes are analysed. The constraint is obtained from the formula of the shadow for Schwarzschild-de Sitter black hole as compared with observational data of the supermassive black hole M87\*:

$$1 + \frac{\mathbb{E}(r_{sh})}{r_{sh}} \geq \frac{r_{sh,\Lambda}}{r_{sh,sh}} = \frac{1}{\sqrt{1 - 9 \left( \frac{GM}{c^2} \right)^2 \Lambda}}$$

This relation, together with data on the radius of the M87\* black hole, makes it possible to find a constraint on the cosmological constant. In addition, one more limitation can be found based on the data for the mass of the black hole itself. We get the following two constraints. Based on shadow data

$$\Lambda \leq 1.542 \times 10^{-28} [m]^{-2}$$

And based on the data for the mass

$$\Lambda \leq 2.214 \times 10^{-28} [m]^{-2}$$

Within the parametrized post-Newtonian (PPN) formalism from the shadow data the value for the parameter  $\zeta$  is obtained

$$\zeta_{\Lambda} = 4.77 \times 10^{-26}$$

The value of  $\zeta$  is a measure of deviations from General Relativity and also is a constraint on various modified theories of gravity.

In this chapter also considers the influence of the cosmological constant on the structure and dynamics of the accretion disk. Since the cosmological constant enters both the relativistic and the weak field limits, in the first case it contributes to the relativistic precession, and in the second case to the Lense-Thirring precession. Taking into account these additions in the fluid equations, which describe the motion of the accretion disk, in addition, taking into account the additional term in the gravitational potential, one can find the nodal and apsidal precession frequencies for the accretion disk. They are respectively written in the following form

$$\Omega^2 = -\frac{6S}{r^4 \omega_{p-K,\Lambda}^2} \sqrt{\frac{GM}{r} - \frac{\Lambda c^2 r^2}{3}}$$

$$\kappa^2 = 4\Omega^2 - \frac{9}{r^5} \frac{\frac{2\Lambda c^2 r^3}{3} - 3GM}{\sqrt{\frac{GM}{r} - \frac{\Lambda c^2 r^2}{3}}}$$

where  $\omega_{p-K,\Lambda}$  is the modified post-Keplerian frequency.

### Chapter 3

The third chapter is devoted to the use of action variable technique and the adiabatic invariant quantities for the Schwarzschild-de Sitter metric. The corresponding action variable expressions are found

$$J_r = J_r^0 + \frac{4M^2\Lambda}{3}J_r^1 + o(\Lambda)$$

where  $J_r^0$  and  $J_r^1$  are expressed in terms of the Weierstrass  $\wp$  functions by following way:

$$J_r^0 = 4\omega L \left[ -1 + \wp'(b)Q(b) + \frac{1}{\wp'(a)} \left( \frac{\wp''^2(a)}{\wp'^2(a)} - \wp'^2(b) \left( 1 - \frac{\wp''(a)}{\wp'^2(a)} \right) \right) Q(a) - \left( \frac{\wp'^2(b)}{\wp'^2(a)} + \frac{\wp''(a)}{\wp'^2(a)} \right) S(a) \right];$$

$$J_r^1 = 2\omega L \left[ 2S(b) + \frac{4\wp'^2(b) - \wp''(a)}{2\wp'^2(a)} + \frac{5\wp''(a)(-2\wp'^2(b) + \wp''(a))}{6\wp'^6(a)} + \frac{Q(b)(-1 + 8\wp'^2(b) - \wp''(a) + 2\wp''(b))}{\wp'(b)} + \frac{1}{6\wp'^6(a)} S(a) \left( 6\wp'^4(a)(-1 + 6\wp'^2(b) - \wp''(a)) + 9\wp'(a)\wp''(a)(-4\wp'^2(b) + \wp''(a)) + (2\wp'^2(b) - \wp''(a))(15\wp'^2(a) - 4\wp'^3(a)) \right) + \frac{1}{6\wp'^7(a)} Q(a) \left( 15(2\wp'^2(b) - \wp''(a))\wp'^3(a) + 6\wp'^6(a)(1 - 8\wp'^2(b) + \wp''(a)) - 6\wp'^4(a)\wp''(a)(1 - 6\wp'^2(b) + \wp''(a)) + 9\wp'^2(a)\wp''(a)(-4\wp'^2(b) + \wp''(a)) + 3\wp'^3(a)(4\wp'^2(b) - \wp''(a))\wp'^3(a) + \wp'(a)(2\wp'^2(b) - \wp''(a))(10\wp''(a)\wp'^3(a) + \wp'^4(a)) \right) \right].$$

The formulae for orbital parameters, the eccentricity and focal parameter, in the case of the Kepler problem for the modified gravity potential are obtained. For example, it is shown that, the eccentricity ceases to be a combination of only action variables, which, in its turn implies that, the eccentricity to be a conserved quantity (in the adiabatic sense).

$$e^2 \approx 1 - \nu^2 + \frac{M^2}{I_\phi^2} (\nu^2 + 3\nu - 4) \nu^2 + \frac{I_\phi^4}{M^4 \nu^4} [I_\phi^2 (3\nu^2 - 5) + M^2 (20\nu^4 + 27\nu^3 - 140\nu^2 - 75\nu + 190)] \frac{\Lambda}{6},$$

This difference is demonstrated in numerical simulation (Fig. 3), where it is shown that if the mass of the central body changes adiabatically, then in the case of Kepler the eccentricity does not change (in the adiabatic sense), but in the case of the appearance of an additional term related to the cosmological constant, eccentricity decreases linearly.

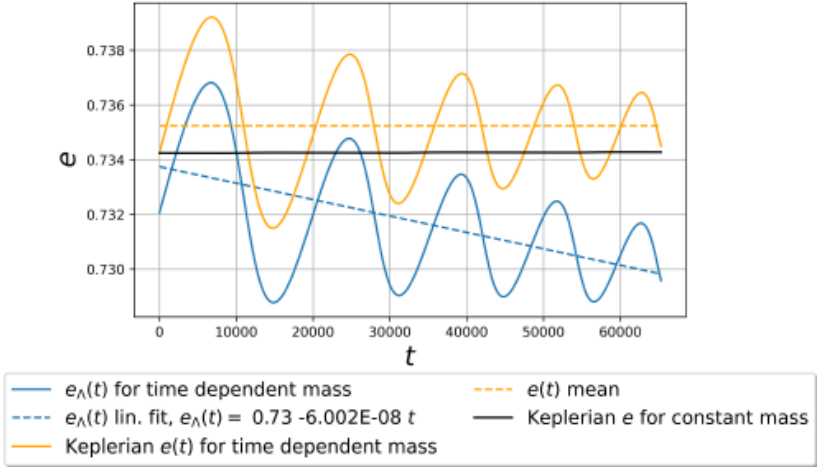


Figure.3 The variation of eccentricities for Newtonian  $e(t)$  and  $\Lambda$ -potentials  $e_\Lambda(t)$  for the given input parameters.

In this chapter also considered the tidal disruption effect. The effect of tidal disruption of stars near the massive black holes is studied in modified gravity case, the formulae for the main parameters are derived and certain increase of the star flux to the loss cone is shown.

$$f_{\Lambda}^{full}(r) = (2\pi)^{1/2} \frac{\Gamma(\alpha + 1)}{\Gamma(\alpha - 1/2)} \times \nu_0 \times \left( \frac{GM_{BH}}{2r} + \frac{\Lambda c^2 r^2}{3} \right)^{\alpha-3/2} \times \left( \frac{1}{\sigma_h^2} \right)^{\alpha} \times \left[ 2GM_{BH}r_{t,\Lambda} + \frac{\Lambda c^2}{3} r_{t,\Lambda}^4 \right]$$

$$f_{\Lambda}^{full}(10^{14}) = 1.671920 \times 10^{-3} [\text{yr}]^{-1}$$

$$f_{\Lambda=0}^{full}(10^{14}) = 1.671918 \times 10^{-3} [\text{yr}]^{-1}.$$

As it is clear, the value of  $f^{full}$  becomes larger for  $\Lambda$ -gravity than for Newtonian case, which means that the contribution of  $\Lambda$  leads to more stars to be torn apart. The presence of repulsive  $\Lambda$ -term affects the nature of dynamics of the orbiting stars, especially at larger radii, the notions and concepts of the tidal radius and tidal force.

It is shown that due to high matter density and hence the decrease of the the Roche limit (Table 3), the pulsars can cross the black hole horizon without being affected by tidal disruption, thus they can be used as unique clocks to trace the metric near black holes' horizon.

Star	Density [kg][m] <sup>-3</sup>	Compare $\left(\frac{3\Lambda c^2}{G}/4\pi\rho_\star\right)$	$r_{t,\Lambda}$ [m]
PSR J0348+0432	$4.293542 \times 10^{17}$	$8.327896 \times 10^{-44}$	$1.300959 \times 10^6$
Sun	1403.817238	$2.547068 \times 10^{-29}$	$8.765271 \times 10^{10}$
Sirius	578.175614	$6.184310 \times 10^{-29}$	$1.178100 \times 10^{11}$
Procyon	225.8598	$1.583114 \times 10^{-28}$	$1.611595 \times 10^{11}$
Pismis 24-17	22.287349	$1.604326 \times 10^{-27}$	$3.487515 \times 10^{11}$
R136a1	5.010613	$7.136088 \times 10^{-27}$	$5.735508 \times 10^{11}$
Aldebaran	$1.787027 \times 10^{-2}$	$2.000875 \times 10^{-24}$	$3.753979 \times 10^{12}$

Table 3

## Conclusion

In Conclusions, the main results of the dissertation are listed:

1. The Lense-Thirring is studied for the Kerr metric, that is in strong field approximation, the appearance of an additional effect, the nutation is revealed.
2. Expressions for the precession rate in the weak field limit – Lense-Thirring metric -- and in the strong field limit, are obtained, which show that, the for weak field modified gravity with cosmological constant, the additional term cannot be interpreted as a matter of normalized density. It is shown that, within the framework of the gravito-electromagnetic formalism, an additional term in the Lense-Thirring precession does appear, leading to a correction in the gravito-gyromagnetic ratio.
3. Observational constraints using the Lense-Thirring effect are obtained on the cosmological constant term using empirical data of LAGEOS and LARES satellites.

4. It is shown that the data for the observed shadow of the supermassive black hole in the galaxy M87\* are in agreement with the modified expression for the shadow radius for the Schwarzschild-de Sitter metric.
  
5. For the Schwarzschild-de Sitter metric and using the Weierstrass elliptic functions, expressions are derived for the action variables and of adiabatically invariant quantities. In the weak field limit, unlike the Keplerian case, in the modified gravity case, the eccentricity of a finite orbit ceases to be a simple combination of adiabatically invariant action quantities, and therefore, the corresponding external and internal parameters of the system are not preserved (in the adiabatic sense).
  
6. It is shown that the repulsive cosmological constant term tends to increase the tidal disruption rate of stars near a massive black hole. For pulsars, due to their high matter density, the Roche (tidal) radius can reach the radius of the horizon of galactic massive black holes. This implies, that the pulsars can cross the black hole's horizon without being tidally destroyed, thus revealing their role as of unique clocks to trace the time dilation and the space-time metric.

### **Publications list**

1. A. Stepanian, Sh, Khlghatyan, V. G. Gurzadyan. "Tidal disruption effects near black holes and Lambda-gravity". *The European Physical Journal Plus* 137, 965 (2022).
2. S. Khlghatyan, A.A. Kocharyan, A. Stepanian, V. G. Gurzadyan. "The cosmological constant vs adiabatic invariance." *The European Physical Journal Plus*, 137, 458 (2022).
3. A. Stepanian, S. Khlghatyan, V.G. Gurzadyan. "Black hole shadow to probe modified gravity". *The European Physical Journal Plus*, 136, 127 (2021).
4. S. Khlghatyan. "On the role of  $\Lambda$  on accretion disks". *The European Physical Journal Plus*, 136, 456 (2021).
5. A. Stepanian, S. Khlghatyan, V.G. Gurzadyan. "Lense–Thirring precession and gravito–gyromagnetic ratio". *The European Physical Journal C*, 80, 1011, (2020).
6. A. Stepanian, S. Khlghatyan. "Lense–Thirring precession and modified gravity constraints". *The European Physical Journal Plus*, 135, 712, (2020).
7. V.N. Stokov, S. Khlghatyan. "The orbital Lense–Thirring precession in a strong field". *General Relativity and Gravitation*, 51, 82 (2019).



## **Լենզե-Թիրինգի երևույթը և մոդիֆիկացված գրավիտացիա**

### **Ամփոփագիր**

Ատենախոսությունը նվիրված է Հարաբերականության ընդհանուր տեսության թույլ դաշտի մոդիֆիկացված տարբերակի շրջանակներում Լենզե-Թիրինգի երևույթի ուսումնասիրությանը, դիտարկելով նաև առնչվող երևույթներ: Այդ մոդիֆիկացված գրավիտացիայի մոտեցումով բացատրվում են դիտողական տվյալներ գալակտիկաների խմբերի և կույտերի, գրավիտացիոն ոսպնյակների, գալակտիկաների պտույտի վերաբերյալ: Լենզե-Թիրինգի երևույթը դիտարկված է ուժեղ դաշտերի մոտավորությամբ, արտածված են համապատասխան բանաձևերը: Լենզե-Թիրինգի մոդիֆիկացված տարբերակի վրա սահմանափակումներ են ստացվել օգտագործելով արբանյակային տվյալներ: Գրավիտո-էլեկտրամագնիսականության մոտեցման շրջանակներում արտածված է Լենզե-Թիրինգի մոդիֆիկացված պարամետրը: Ուսումնասիրված են սև խոռոչի անմիջական շրջակայքի երևույթներ՝ նրա ստվերը և ակրեցիոն սկավառակը: Շվարցշիլդ-դե Սիտեր սև խոռոչի ստվերի համար արտածված համապատասխան բանաձևերը համեմատվել են M87\* գալակտիկայի կենտրոնի գերհսկա սև խոռոչի ստվերի դիտողական տվյալների հետ և ստացվել են սահմանափակումներ կոսմոլոգիական հաստատուն պարունակող անդամի վրա, արտածվել են նաև պարամետրացված պոստ-նյուտոնյան ֆորմալիզմի (PPN) պարամետրերը:

Վեյերշտրասի էլիպտիկ  $\wp$  ֆունկցիայի հիման վրա Շվարցշիլդ-դե Սիտեր մետրիկայի համար արտածվել են ադիաբատիկ ինվարիանտ մեծությունների՝ գործողության փոփոխականների արտահայտությունները, մասնավորապես, ցույց է տրված, որ էքսցենտրիցիտը դադարում է լինել ադիաբատիկորեն պահպանվող մեծություն: Հետագոտված է աստղերի մակընթացային քայքայման երևույթը մեծ զանգվածով սև խոռոչների շրջակայքում գալակտիկաների միջուկներում, ներառյալ կոսմոլոգիական հաստատունի ներդրումը աստղերի հոսքում:

## **Эффект Лензе-Тирринга и модифицированная гравитация**

### **Резюме**

Диссертация посвящена изучению эффекта Лензе-Тирринга в рамках Общей теории относительности (ОТО) с модифицированным слабым полем с космологической постоянной, а также рассмотрены ассоциированные явления. Рассмотрен эффект Лензе-Тирринга в сильном поле, т.е. в метрике Керра, и показано, кроме прецессии, также наличие нутации. В рамках подхода гравито-электромагнетизма показано наличие дополнительного члена связанного с космологической постоянной. Исследованы явления в непосредственной близости черных дыр, а именно тень черной дыры и аккреционный диск с модифицированным полем. Используя модифицированное выражение для радиуса тени черной дыры Шварцшильда-де Ситтера и наблюдательные данные для тени супермассивной черной дыры в центре галактики M87, получены ограничения на отклонения от ОТО, включая в рамках пост-ньютоновского формализма (PPN). Получены формулы для метрики Шварцшильда-де Ситтера для адиабатически инвариантных величин - переменных действия, выраженные через эллиптическую функцию Вейерштрасса  $\wp$ , в частности, показано, что эксцентриситет перестает быть адиабатически инвариантной величиной. Исследовано явление приливного разрушения звезд вблизи массивных черных дыр в центрах галактик для случая модифицированной гравитации, выведены формулы для основных параметров процесса, и показано, что космологический член увеличивает поток звезд в конус потерь.