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Linz, May 14th, 2025

Report on the thesis
"Basis properties for orthogonal spline systems"

It is my pleasure to give a report on the thesis "Basis properties for orthogonal spline systems", submitted by Levon Aram Hakobyan in fulfillment of the requirement for the PhD in Physical and Mathematical Sciences, Specialization: A.01.01-Mathematical Analysis.
The thesis at hand consists of four chapters, based on the four works [57, 58, 59, 60] by the author as referenced in the thesis. It is summarized in the attached synopsis, which corresponds to the underlying dissertation. The first three chapters consider sufficient and necessary conditions under which periodic orthogonal spline systems are bases and unconditional bases in the corresponding periodic Hardy space $H^1(\mathbb{T})$. The fourth chapter proves results about when the coefficients of certain orthogonal tensor spline series can be recovered by their limiting functions.
To be more precise, Chapter 1 proves that orthonormal spline functions (f_n) of order k on the torus $\mathbb{T}$ form a Schauder basis in $H^1(\mathbb{T})$ if and only if in every step of the refinement of the mesh, ratios of lengths of neighbouring B-spline supports of order k are bounded from above and below uniformly by some positive constants. (This geometric property of the mesh is called *k-regularity*).
Chapters 2 and 3 prove the necessity and sufficiency of the stronger geometric condition of $(k-1)$-regularity for (f_n) to be an unconditional basis in $H^1(\mathbb{T})$.
Chapter 4 contains two results of similar nature, one of which reads as follows: If $E \subset [0,1]^d$ is a product of d Lebesgue zero sets, the pointwise convergence of an orthogonal tensor spline series on the complement of E to a finite integrable function f implies that the coefficients of the series can be reconstructed from the limit function f.
The topics of Chapters 1 through 3 are, in my opinion, very natural extensions of different existing results in that direction by G. Gevorkyan, A. Kamont, K. Keryan and M. Passenbrunner. In particular, an analogous result was proved for orthonormal spline systems on the interval by G. Gevorkyan, A. Kamont, K. Keryan and M. Passenbrunner and the analysis of the unconditional basis property of orthonormal spline systems in L^p-spaces on the torus by K. Keryan and M. Passenbrunner provided estimates that are useful in the underlying dissertation.
The results contained in this thesis are new and interesting. Their proofs are correct and show a remarkable command of the necessary analytical toolbox by the author. The dissertation is clearly written, well structured, and preluded by a carefully thought-out introduction that summarizes earlier works in this direction. In conclusion, it is my opinion that Levon Aram Hakobyan deserves to be awarded the PhD in Physical and Mathematical Sciences, Specialization: A.01.01-Mathematical Analysis.

Sincerely,

INSTITUT FÜR ANALYSIS Universität Linz

Markus Passenbrunner