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The referee report on the Dissertation  
Periodic Groups, their Automorphisms and Probabilistic Identities in Groups  
by Arman A. Bayramyan  
Speciality: A.01.06 "Algebra and Number Theory"

It is my pleasure to write a report on the Ph.D. thesis "Periodic Groups, their Automorphisms and Probabilistic Identities in Groups" by Arman A. Bayramyan. The manuscript under consideration is devoted to a deep and actively developing research direction close to the well-known Burnside problem, namely the problem of finiteness of a finitely generated torsion group.

Group theory is one of the rapidly and actively developing branches of mathematics, strongly connected with other areas of mathematics, physics, and chemistry. The class of problems considered in this work is of high theoretical and practical interest. The relevance of the dissertation topic is determined both by the wide range of important applications of results obtained in this area and by the existence of numerous open problems, including the Burnside problem.

The work under consideration addresses the growth of free Burnside groups, the relationship between probabilistic and universal fulfillment of group identities, and a fixed-point problem for automorphisms of free Burnside groups. The main results of this work are published in 3 papers in highly reputed journals. All results presented in the work are important, interesting, and possess with the mathematically correct proofs. Among all the results I would like to underline the following four main statements which seem to be the most inspiring from my point of view:

1. Establishing of an explicit lower bound for the growth function of free Burnside groups of an arbitrary rank  $m \geq 2$ , which extends of a well-known result by Adian.
2. Explicit growth estimates for the broad class of finitely generated  $n$ -torsion groups.
3. Construction of an explicit counter-example to the known conjecture by Amir, Blachar, Gerasimova, and Kozma concerning the relations between probabilistic and universal identities in groups. Namely, a construction of a continuum family of non-isomorphic 3-generator groups all satisfying  $x^n = 1$  with probability 1, while the identity fails universally in each of the groups.
4. Reformulation of Mazurov's problem on fixed points for automorphisms of free Burnside groups in cohomological setting.

The obtained results represent a major new scientific achievement in geometric group theory. Open problems have been resolved, and new original methods have been proposed.

This work consists of 7 chapters. Let's review the contents of the work chapter by chapter.

The first chapter is of introductory nature and contains nice historical survey of the subject and general description of the achieved results.

In the second chapter the author presents an extensive overview of the theory of periodic groups, including,  $n$ -periodic products and  $n$ -torsion groups. This chapter contains the constructions and their properties that the author uses later.

The third chapter contains preliminaries and necessary results on the geometric group theory.

In Chapter 4 the author describes the probabilistic concepts necessary for formulation and proof of his main results 1 and 2, namely, to provide an explicit lower bound for the growth function of free Burnside groups of arbitrary rank.

Chapter 5 contains the proofs of the main results concerning probabilistic identities, in particular, the counter-example in the item 3 above.

In Chapter 6 the author studies the automorphisms of free Burnside groups, in particular, reformulates Mazurov's problem on fixed points of automorphisms in cohomological setting.

The last chapter is concluding. It contains the summary of the results as far as the open problems and further research directions.

Thus, the manuscript under consideration proposes a completely new method for solving a series of important and relevant problems in geometric group theory which demonstrate in particular its scientific novelty. The practical significance of the work lies in its complete and comprehensive study of the theoretical issues under consideration. The results obtained are new, relevant, and reliable; the main results are published in three papers in refereed journals. The system of theoretical propositions formulated and proved by the author undoubtedly represents a new scientific achievement in geometric group theory and is a nice advancement in its field.

The content of the dissertation corresponds to the specialty for which it is recommended for defense. The dissertation by Arman A. Bayramyan, "Periodic Groups, their Automorphisms and Probabilistic Identities in Groups" is, overall, a complete scientific achievement and meets all necessary requirements for the Ph.D. Degree. The author of the manuscript deserves the degree of Candidate of Physical and Mathematical Sciences, specialization A.01.06 — Algebra and Number Theory.

Sincerely

Professor  
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