

OFFICIAL OPPONENT'S REVIEW

of the thesis in the form of a scientific report

“Exactly Solvable Non-Equilibrium Lattice Models, Algorithms, and Schemes for Optimal Information Dissemination in Self-Organizing Multi-Agent Systems” by Vahagn S. Poghosyan, submitted for obtaining doctoral degree in technical sciences in specialty 05.13.05 “Mathematical modeling, numerical methods and program complexes”

Nonequilibrium models are theoretical concepts used in physics, mathematics, economics, and ecology to describe systems that constantly exchange energy or matter with their environment and lack thermodynamic equilibrium. Unlike traditional theories that assume perfect balance, these models track temporal evolution, irreversible processes, and continuous environmental interactions. The spin models are important not only in their classical lattice formulations, such as the Potts model, but also in the theory of quantum spin chains, including the Heisenberg and XXZ-type models. Mathematically, these models are closely related to non-equilibrium exclusion processes, whose Markov generators can be represented in the quantum Hamiltonian formalism. This connection explains the effectiveness of the Bethe ansatz in the analysis of totally asymmetric simple exclusion process (TASEP) and related interacting-particle systems.

The thesis in the form of a scientific report entitled “Exactly Solvable Non-Equilibrium Lattice Models, Algorithms, and Schemes for Optimal Information Dissemination in Self-Organizing Multi-Agent Systems” by Vahagn S. Poghosyan presents precisely a broad and scientifically well-motivated study at the intersection of statistical mechanics, discrete (lattice) mathematics, graph theory, computational algorithms, and applications to self-organizing autonomous multi-agent systems. The work summarizes significant results obtained by the author and published in a large number of scientific articles. The thesis has an interdisciplinary character and connects exact analytical methods in lattice models with algorithmic constructions for optimal information dissemination and with applied models of decentralized coordination in unmanned aerial vehicle swarms.

The first part of the thesis is devoted to the state of the art and to the author's main results in equilibrium and non-equilibrium statistical mechanics. This part provides a systematic overview of exactly solvable lattice models, including the Ising model, Potts model, dimer model, Abelian sandpile model, loop-erased random walks, uniform spanning trees, the $O(n)$ loop model, the rotor-router model, and the totally asymmetric simple exclusion process. The author clearly explains the role of exact methods, combinatorial representations, transfer matrices, determinant techniques, and numerical simulations in the study of these models.

An important group of results concerns the Potts model, where the author studied the q -state Potts spin model on anisotropic ladder lattices. The representation of the partition function as a sum over subgraphs was used together with the deletion-contraction property, leading to recurrent systems of linear equations and fixed-size transfer matrices. This allowed the exact computation of partition functions and the construction of Fisher zeros in the complex plane.

A substantial part of the first section is devoted to the totally asymmetric simple exclusion process (TASEP). The author investigated discrete-time TASEP on finite rings with a fixed number of particles and with different update rules. Exact expressions were obtained for transition probability generating functions and mean current velocities. In particular, explicit formulas for the stationary velocity were derived for arbitrary ring size, number of particles, and hopping probability, and the thermodynamic and continuous-time limits were shown to reproduce known results. The author also considered more general cases with particle-dependent hopping probabilities and obtained determinant formulas for non-stationary transition probabilities by using a combinatorial Bethe ansatz approach.

Another important contribution concerns the close-packed dimer model with a single monomer on the two-dimensional square lattice. By using the spanning webs representation, which generalizes the spanning tree approach, the author derived determinant expressions for random variables describing dimer mobility. In the large-lattice limit, these expressions were reduced to Toeplitz determinants and their minors. Exact values were obtained for the probability of monomer mobility and related diffusion characteristics, confirming previously proposed numerical conjectures.

The results related to the Abelian sandpile model are among the central achievements of the first part of the thesis. The author studied the two-dimensional Abelian sandpile model on the square lattice and its connection with uniform spanning trees, loop-erased random walks, and close-packed monomer-dimer models. It was shown that the mean height in the Abelian sandpile model is directly related to the return probability, or looping constant, of the loop-erased random walk. The exact value of this probability was obtained, leading to the exact rational value of the mean height. These results resolved earlier conjectures concerning height probabilities and clarified the remarkable fact that, although the individual height probabilities contain irrational expressions, the mean height is rational.

The thesis also presents results on two-point correlation functions of height variables in the Abelian sandpile model. In particular, asymptotic expressions for correlations involving height 1 were obtained by combinatorial methods, confirming predictions of logarithmic conformal field theory with central charge $c=-2$. This connection between the Abelian sandpile model and logarithmic conformal field theory is of considerable theoretical importance, since it reveals the non-local structure underlying local observables in self-organized critical systems.

The author further considered the fixed-energy sandpile model and its relation to the stationary density of the Abelian sandpile model with open boundaries. A conjecture was proposed concerning the convergence of the threshold density to the stationary density in the limit of large negative initial height. This conjecture was later proved in a mathematical work by Levine, which confirms the significance of the author's contribution to the development of the theory.

The first part also contains results related to percolation and spanning tree models on the half-plane, correlations of multiple loop-erased random walk trajectories, the dimer model on a cylinder, and the rotor-router model. For the rotor-router model, the author obtained results on correlations between dimers and contours, loop reversibility, subdiffusive behavior, spiral structures on the infinite square lattice, and rotor-router walks on a semi-infinite cylinder. These

results contribute to the mathematical understanding of deterministic analogues of random walks and their relation to self-organized criticality.

From my point of view, the first part is of particular importance, as it contains substantial theoretical results that form an essential foundation for the subsequent developments presented in the thesis.

The second part of the thesis is devoted to a *mathematical* topic, namely the theory of optimal information exchange. This direction is based on the theory of gossip and broadcast schemes, Knödel graphs, fault-tolerant communication, ascending paths, and graph-theoretic models of full information dissemination. The author's results in this part are connected with the construction of optimal and fault-tolerant gossip schemes, the method of local interchange, edge-permuted and modified Knödel graphs, and the development of software tools for the investigation of such schemes. The topic is mathematically relevant because it addresses the optimization of the number of calls, the number of rounds, the number of communication links, and the degree of fault tolerance in distributed networks.

The third part of the thesis is devoted to *applications* of the author's theoretical and algorithmic results to self-organizing unmanned aerial vehicle swarms. These systems are realized as autonomous self-organizing multi-agent systems, in which individual agents interact locally, exchange information, coordinate their behavior, and perform collective tasks without centralized control. The application of optimal information exchange schemes, fault-tolerant gossip/broadcast algorithms, rotor-router and multi-Eulerian coverage methods, and cloud-based software platforms to unmanned aerial vehicle swarms demonstrates the practical relevance of the theoretical results obtained in the previous parts of the thesis. The applied part is especially important because it connects abstract mathematical models with modern technological systems. The proposed approaches support decentralized coordination, robust information dissemination, task execution, numerical modeling, simulation, and area coverage by autonomous unmanned aerial vehicle swarms. In this sense, the thesis not only contributes to the theory of exactly solvable lattice models and information dissemination schemes, but also provides a basis for practical algorithmic solutions in distributed cyber-physical systems.

Suggestions and Remarks:

- It would have been appropriate to include a reference to Lindblad's seminal work [G. Lindblad, Commun. Math. Phys. 48 (1976) 119], which laid one of the fundamental foundations for the description of nonequilibrium quantum open systems and has had a major influence on the subsequent development of this field.
- Given the significant role of quantum dynamical semigroups in the theory of non-equilibrium and open quantum systems, it would have been appropriate to include a discussion of the fundamental work [Kraus, K.: Ann. Phys. 64, 311—335 (1971)]. In particular, the thesis could have more clearly addressed the connection between the considered non-equilibrium models and the general formalism of completely positive dynamical maps and quantum Markovian evolution.

The thesis is well structured and reflects a long-term research program. Its scientific novelty lies in the combination of exact analytical results in non-equilibrium lattice models, combinatorial

and graph-theoretic methods, numerical modeling, and applications to autonomous multi-agent systems. The author demonstrates a high level of mathematical and computational competence and shows the ability to connect deep theoretical results with applied algorithmic constructions.

Vahagn Poghosyan has collaborated with such distinguished scientists as V. Priezzhev and P. Ruelle. Among the numerous articles published jointly with these outstanding researchers, the following three works may be particularly highlighted [J Stat Mech: Theory and Experiment, P07025 (2010); J Stat Mech: Theory and Experiment, P10004 (2011); Phys Lett B 659, 768 (2008)].

The main results of the thesis have been published in peer-reviewed scientific papers, which confirms their scientific validity and relevance. The presented results are of interest to specialists in statistical mechanics, discrete mathematics, graph theory, distributed algorithms, and multi-agent systems.

The thesis satisfies the requirements for a thesis submitted in the form of a scientific report for the degree of Doctor of Science.

Vahagn S. Poghosyan's thesis in the form of a scientific report entitled "Exactly Solvable Non-Equilibrium Lattice Models, Algorithms, and Schemes for Optimal Information Dissemination in Self-Organizing Multi-Agent Systems" to be a completed scientific work that makes a significant contribution to the mathematical modeling of lattice processes, their theoretical and numerical investigation, and the development of methods for algorithmic support of self-organizing systems. The content of the thesis corresponds to the specialty 05.13.05 "Mathematical Modeling, Numerical Methods and Program Complexes".

I therefore support the submission of this thesis for defense and consider that Vahagn S. Poghosyan deserves to be awarded the degree of Doctor of Science.

Official Opponent

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I hereby certify the signature of N. S. Ananikyan.

5.10.26

